

Intertemporal Decision-Making and Loss Aversion

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Xiao Qiao, Feb 2008

Declaration

I hereby declare that this PhD thesis is my own work and that all other works discussed and referred to have been duly cited, and the work has not been submitted for any other degree or professional qualification.

Abstract

This thesis is devoted to a novel approach to intertemporal decision-making. Specifically, it applies the reference-dependent model to intertemporal choices and shows that one can model intertemporal choice even without the assumption of time discounting.

In Chapter I, I present a brief review of the background literature, which could be of help for our understanding of intertemporal decision-making. Specifically, it focuses on three prominent economic theories, which are the most relevant for my work on intertemporal decision-making, namely, the exponential discounted utility theory, the hyperbolic discounting theory, the theory of reference-dependent preferences, and the similarity-based procedure.

In Chapter II, I present the intertemporal reference-dependent model composed of an intrinsic consumption utility and a reference-dependent gain-loss utility which are additively separable over time. Three simple ways of reference point determination – choice-set dependent, choice-set independent, and hybrid reference points are also explored. In addition, I also argue that the intertemporal reference-dependent model may offer an alternative explanation of the experimental observations by Rubinstein (2003). Furthermore, in Chapter III, I extend the intertemporal reference-dependent model, where an individual is uncertain about her future circumstances, and her reference levels follow a random walk process. While the model does not explicitly include time discounting and return on saving, it nevertheless may offer an alternative explanation of present bias and negative time preference (future bias), and may also offer an alternative explanation of dynamically inconsistent preferences over time.

Finally, in Chapter IV, using a survey-based within-subject choice experiment, I explore whether subjects' behaviour is consistent with a number of existing and emerging theories, namely hyperbolic discounting theory, Rubinstein's similarity procedure, and the novel intertemporal reference-dependent model. The main result of the survey-based within-subject choice experiment is that none of these three theories explain subjects' behaviour. However, I do find that, among these three theories, the novel intertemporal reference-dependent model performs no worse than Rubinstein's similarity procedure, and hyperbolic discounting theory performs the worst with subjects' behaviour on subset of questions. Moreover, I could reject the hypothesis that the subjects made their choices at random. Finally, I also found that their behaviour is not consistent with the Independence of Irrelevant Alternatives.

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An Introduction to Intertemporal Decision-Making and Loss Aversion

Abstract: In this chapter, I provide a brief review of the background literature, which could be of help for our understanding of intertemporal decision-making. Here, I review three prominent economic theories, which are the most relevant for my theoretical and experimental work on intertemporal decision-making, namely, the exponential discounted utility theory, the hyperbolic discounting theory, the theory of reference-dependent preferences, and the similarity-based procedure. Aim of this chapter is to provide motivation and background for my thesis on intertemporal decision-making and loss aversion.

1. Introduction

The growing body of experimental research suggests that neoclassical theory of individual choice is at odds with observed human behaviour. Being one of the most important human activities, intertemporal decision-making has been of a particular interest to economists for at least one hundred years and the exploration of intertemporal decision-making could be dated back to as early as Adam Smith. Despite this long history of research on decision-making over time, economists still disagree about what is the best way to model it.

In this chapter, I give a brief review of three prominent economic theories, which could be of help for our understanding of intertemporal decision-making, and which are the most relevant to my work on intertemporal decision-making. I begin with the review of the main features and the criticism of the exponential discounted utility theory (due to Samuelson, *Review of Economic Studies*, 1973), which is the main neoclassical theory of intertemporal choice. I next review the most prominent alternative to this theory, which is the hyperbolic discounting theory and was widely popularized by Laibson (*QJE*, 1997). After that, I present the theory of reference-dependent preferences due to Kahneman and Tversky (*Econometrica*, 1979), which is central to my theoretical model of intertemporal decision-making.

Finally, I present the similarity-based procedure due to Rubinstein (JET, 1988), which is important for my experimental work. This review is aimed to provide motivational background for my thesis on loss aversion in intertemporal decision-making.¹

The earliest approach to intertemporal decision-making behaviour involved a combination of different psychological motives, which either promote or suppress desires for immediate consumption. In the first half of the 20th century, the observed intertemporal decision-making behaviour was modelled using the concepts of the marginal rate of substitution of consumption bundle over time, and of diminishing marginal utility.

In his ground-breaking work, “A Note on Measurement of Utility” (1937), Samuelson introduced a parsimonious model of intertemporal preferences by compressing all psychological factors underlying intertemporal choice into the discount rate. Since then, the exponential discounted utility model has dominated economic theory of intertemporal decision-making. Although the exponential discounted utility model has been applied to analyse intertemporal decision-making in a variety of situations, including savings behaviour, labour supply, etc., it has been

¹ Here I focus on some important works and the most recent studies covering two areas that are relevant to my thesis - namely, intertemporal choice anomalies and loss aversion. I thus leave out the theoretical models that involve modifications of the instantaneous utility function, such as habit-formation models, utility-from-anticipation models, visceral-influences models, projection-bias models, etc. (for discussion on these alternative models see Frederick, Loewenstein, and O'Donoghue (2002)).

found to be inconsistent with both empirical and experimental findings on intertemporal choice by cognitive psychologists and economists. A few insightful empirical and experimental studies criticised the exponential discounted utility theory and have led to a number of alternative theoretical models.

This chapter is organized as follows. The exponential discounted utility theory and its criticism are discussed in Section 2; while three alternative theoretical models are discussed in subsequent sections. Section 3 is devoted to the hyperbolic discounting theory, which captures the observation that individuals' preferences are inconsistent over time; Section 4 presents the theory of reference-dependent preferences, which captures the observation that individuals evaluate consumption bundles relatively to some reference points; and Section 5 describes similarity procedure, which fundamentally departs from framework of the exponential discounted utility model. Finally, Section 6, I discuss how the present thesis fits into the existing literature.

2. The Exponential Discounted Utility Model and Its Criticism

In this section, I introduce the exponential discounted utility model, and present

several empirical and experimental observations, which question the validity of the exponential discounted utility model.

2.1. The Exponential Discounted Utility Model

The exponential discounted utility model of Samuelson (1937) states that a decision maker's intertemporal preferences over a sequence of consumption profiles, (c_t, \dots, c_T) , could be represented by an intertemporal utility function:

$$U(c_t, \dots, c_T) = \sum_{k=0}^{T-t} \left(\frac{1}{1+\rho} \right)^k u(c_{t+k}),$$

where $u(\bullet)$ is the person's cardinal instantaneous utility function, and ρ is the discount rate for one period, and captures all psychological factors that motivate the intertemporal decision-making. Under assumptions of completeness, transitivity and continuity, a sequence of consumption profiles, (c_t, \dots, c_T) , will be preferred to another sequence (c'_t, \dots, c'_T) , if and only if the following inequality is satisfied:

$$U(c_t, \dots, c_T) > U(c'_t, \dots, c'_T).$$

A central assumption of the exponential discounted utility model concerns individuals' preferences of adding a new consumption bundle into an exiting consumption plan. For example, when an additional consumption prospect $X(x_t, \dots, x_T)$ is available to be added into an exiting consumption bundle (c_t, \dots, c_T) ,

an individual will accept the additional consumption prospect X if and only if the utility of the aggregate consumption bundle (c'_t, \dots, c'_T) , which is adjusted by the additional consumption prospect X , is higher than the utility of the existing consumption plan (c_t, \dots, c_T) , such that $U(c'_t, \dots, c'_T) > U(c_t, \dots, c_T)$, where (c'_t, \dots, c'_T) is the aggregate consumption bundle.

In addition, the exponential discounted utility model explicitly assumes that an individual's intertemporal preferences over a consumption bundle should be independent from her previous consumption experiences or future consumption prospects.

Furthermore, in the exponential discounted utility model, for example, apples and bananas are explicitly assumed discounted at the same rate. Hence, there is a unitary time preferences across all kinds of consumption. In addition, the discount rate ρ is constant over time. For example, if a consumption bundle (c_t, \dots, c_T) is preferred over another one (c'_t, \dots, c'_T) where $c_t = c'_t$, then the consumption bundle (c_{t+1}, \dots, c_T) should also be preferred over the consumption bundle (c'_{t+1}, \dots, c'_T) . In other words, with the passage of time, an individual's current preferences are confirmed by her previous decision, such that her preferences are consistent over time.

In addition, the discounted utility model also assumes that the cardinal instantaneous utility function $u(\bullet)$ is stationary over time and individuals have positive time preference, which motivates an individual to accumulate consumption towards the present.

2.2. Criticism of the Exponential Discounted Utility Model from Experiment

Over the last two decades, a number of empirical and experimental studies on intertemporal decision-making have documented a large number of inadequacies of the descriptive validity of the exponential discounted utility model. First of all, these studies questioned the positive time preference, and also have documented discount rates are diminishing over time rather than constant over time, which is often referred to as hyperbolic discounting. Furthermore, discount rates differ across different types of intertemporal situations, such that gains are discounted more than losses, small amounts of consumption are discounted more than large amounts of consumption, and sequences of consumption bundles are discounted differently from a single consumption bundle.

A number of researchers have put the assumption of positive time preference in the

questionable position. Koopmans (1960), Koopmans, Peter Diamond, and Richard Williamson (1964), Olson and Bailey (1981) argue that a zero or negative time preference with a positive real interest rate on saving will lead an individual to postpone consumption rather than accelerate consumption towards the present.

Thaler (1981) reported that when subjects are required to specify the amount of money that would make them indifferent between receiving \$15 now and receiving the specified amount of money later, subjects averagely require \$20 for one month waiting, \$50 for one year waiting, and \$100 for ten years waiting, which appeared a declining discount rate. The similar pattern of declining discount rate over time are also be found by a large number of researchers. (see Benzion, Rapoport, and Yagil (1989); Rachlin, Raineri, and Cross (1991); Redelmeier and Heller (1993); Chapman and Elstein (1995); Kirby and Marakovic (1995); Myerson and Green (1995); Chapman (1996); Pender (1996); Kirby (1997)).

In addition, in the terms of the standard exponential discounted utility model, an individual's preferences over two intertemporal options, which are a small and sooner payoff and a large but later payoff, should depend merely on the absolute time interval that separates these two payoffs. In fact, an individual's choice over these two payoffs might change when both of them are delayed by a given time periods. For example, in a study of Thaler (1981), an individual might prefer one apple today

to two apples tomorrow, but at the same time she might prefer two apples in 51 days to one apple in 50 days. Therefore, an individual's intertemporal preferences are dynamically inconsistent over time. The similar pattern of inconsistent preferences over time was also found by a large number of researchers in humans (see Solnick, Kannenberg, Eckerman, and Waller (1980); Millar and Navarick (1984); Green, Fristoe, and Myerson (1994); Kirby and Herrnstein (1995)) and in animals (see Ainslie and Herrnstein (1981); Green, Fischer, Perlow, and Sherman (1981)).

The assumption of the stationary cardinal instantaneous utility function has been also considered unrealistic, especially in the research on how an individual changes her taste over time. Suranovic, Goldfarb, and Leonard (1999), and O'Donoghue and Rabin (1999a, 2002) discuss the effect of non-stationarities on addictive behaviour. Furthermore, Loewenstein and Angner (2003) discuss various factors that can entail changes in preferences - including appetites, emotions, social influence, and etc. Apart from declining discount rates, many studies on intertemporal decision-making found that gains are discounted more than losses. For example, in a study by Thaler (1981), subjects were asked to imagine to pay a traffic ticket either now or later, and to specify the amount of money they would be willing to pay if they could pay it later. The discount rates observed from this study were significantly lower than the discount rates revealed from answers to comparable questions about monetary gains. More precisely, Thaler (1981) estimated that discount rates for gains were three to

ten times greater than those for losses. Furthermore, Loewenstein (1988) found a similar pattern that gains are discounted more than losses.

In addition, many experimental studies of time preference have also found the diminishing sensitivity in a sense that large monetary outcomes are discounted more than small ones. For instance, subjects in a study of Thaler (1981) were on average indifferent between receiving \$15 immediately and receiving \$60 in a year, and were also on average indifferent between receiving \$250 immediately and \$350 in a year, as well as on average between an immediate \$3000 and \$4000 in a year. Similar results were also found by many other studies (see Loewenstein (1987); Benzion, Rapoport, and Yagil (1989); Redelmeier and Heller (1993)).

Furthermore, Loewenstein (1988) observed an asymmetric preference between speeding up and delaying consumption, as monetary compensation for delaying an expected consumption by a given interval was from two to four times greater than the amount of money subjects would be willing to pay to speed up an unexpected consumption by the same interval. For instance, subjects who expected to receive a VCR immediately would require, on average, a compensation of \$126 to delay the consumption of VCR by a year; on the other hand, those who didn't expect to have the same item for another year would be on average willing to pay \$54 to have it immediately.

2.3. Alternatives to the Exponential Discounted Utility Model

In response to the above-mentioned observations as well as other intertemporal choice “anomalies”, economists have proposed a large number of alternatives to the exponential discounted utility model. Some of these new models, such as hyperbolic discounting theory, attempt to achieve more descriptive validity by modifying the assumption of constant discount rates. The hyperbolic discounting theory will be discussed in detail in Section 3.

Other alternative models borrow insights from the behavioural theory in a static setting and introduce more assumptions of the instantaneous utility function. One of the prominent “static” theories suggests that individual’s preferences are defined over changes of the level of wealth relative to his or her reference level, rather than depend merely on the absolute final level of outcome; and individuals are more sensitive to losses than gains. In other words, decision under risk can be viewed as a choice between “prospects”. Model of reference-dependent preferences could go back to Kahneman and Tversky (1979), and will be discussed in detail in Section 4.

Other studies fundamentally depart from the neoclassical theory of choice by taking

seriously the cognitive imperfections documented by cognitive psychologists. One of such fundamentally different models, Rubinstein's (1988) similarity procedure, will be discussed in detail in Section 5.

3. Hyperbolic Discounting Theory

Probably, the most prominent alternative to the exponential discounted utility model is hyperbolic discounting theory, which concerns one of the fundamental assumptions of the exponential discounted utility model - consistency of an individual's preferences. In response to the above-mentioned experimental studies, hyperbolic discounting theory was introduced in the middle of 20 century. This theory accounts for the diminishing discount rates, which are inconsistent with the exponential discounted utility model involving a sequence of constant discount rates, and reveal that a person would have inconsistent preferences over time.

In the exponential discounted utility model, an individual's preferences are consistent and coherent over time, which are captured by an exponential discounting function with a constant discount rate. However, there are numerous experimental studies in both animal and human behaviour that find that the individual's preferences are

dynamically inconsistent, leading to a possibility that people's preferences might be instead captured by hyperbolic discounting theory, which postulates that an individual has a relatively high discount rate over short time periods and a relatively low discount rate over long time periods.

3.1. Models of the Hyperbolic Discounting Theory

Phelps and Pollak (1968) introduced a simplified functional form of hyperbolic discounting theory to capture experimental results from studies of animal and human behaviour in cognitive psychology. After the influential work of Laibson (1997), hyperbolically discounted utility function has been used to replace the standard exponential discounted utility model in analysing decision-making over intertemporal choices.

Study of hyperbolic discounting theory could be dated back to Strotz (1956). Phelps and Pollak (1968) proposed a quasi-hyperbolic discounting function to capture experimental results from studies of animal and human behaviour in cognitive psychology. Hyperbolic discounting theory predicts that an individual would have a high discount rate in the near future and a relatively low discount rate in the farther future. This phenomena is captured by the following function

$u(x_0, x_1, \dots, x_t, \dots) = v(x_0) + \beta \sum_{t=1,2,\dots} \delta^t v(x_t)$, where the function $v(x)$ is the instantaneous utility function, hence the discount factor in discrete time periods are $1, \beta\delta, \beta\delta^2, \beta\delta^3, \dots$ respectively.

Hence, the hyperbolic discounting theory illustrates a salient inconsistency between human being's current preferences and those in the future. For example, the discount rate between time periods t and $t+1$ is the long-term discount rate from the current view, and it is lower than the discount rate between time periods t and $t+1$ from the perspective of time t , which is labelled as the short-term high discount rate.

In the influential work of Laibson (1997), hyperbolic discounting function with parameters of β and δ , has been used to explore the role of illiquid assets, such as housing. Laibson (1997) suggested that a person could hold her wealth in illiquid assets as an imperfect technology of self-control to protect herself from over-consumption. Furthermore, without the technology of self-control, Laibson (1998) use hyperbolic discounting functions successfully explaining how a person makes a choice between consumption today and saving for tomorrow².

² Angeletos, Laibson, Repetto, Tobacman, and Weinberg (2001), test the hyperbolic discounting theory in the consumption-saving environment by comparing simulated data to real-world data, and conclude that hyperbolic discounting could explain a variety of empirical observations better than exponential discounting, especially about low liquid assets holdings, high credit-card debt, and consumption-income comovement.

O'Donoghue and Rabin (2001) have applied the hyperbolic discounting function, associated with parameters of β and δ , to explore preferences to procrastination, in which a person underestimates the magnitude of her self-control problems in the future and would never stop trying to figure out an better plan. Hence, she would postpone some easy tasks, and eventually, she have to face the accumulated hard task. In addition, O'Donoghue and Rabin (1999a, and 2000), and Gruber and Köszegi (2000) have applied hyperbolic discounting theory, associated with parameters of β and δ , to explain why some individuals tend to consume too much of some harmful addictive products.

Specially, Rubinstein (2003) proposed the following function formulation of the hyperbolic discounting theory:

$$U(c_0, c_1, \dots, c_t) = v(c_0) + \sum_{t=1,2,\dots} \left(\prod_{s=1,2,\dots,t} \delta_s \right) v(c_t),$$

where the function v is the concave instantaneous utility function, and $\delta_s < 1$ is a (weakly) increasing sequence therefore the time discounting factors in discrete time periods are not constant, namely $1, \delta_1, \delta_1\delta_2, \dots, \prod_{s=1,2,\dots,t} \delta_s$. In this functional form the discount rates are (weakly) decreasing over time rather than a constant sequence.

3.2. Rubinstein's Challenge

Although, the hyperbolic discounting theory could successfully explain a variety of empirical observations and experimental results in the literature on consumption and saving, procrastination, as well as consumption of addictive products, and is better than the standard exponential discounted utility model, the validity of hyperbolic discounting theory has also received some challenge.

In a recent study, Rubinstein (2003) introduced three experimental observations, which challenge descriptive validity of not only the standard exponential discounted utility model but the hyperbolic discounting theory with an increasing and concave instantaneous utility function as well.

For example, in the study of Rubinstein (2003), subjects were asked (in the year of 2002) to choose the favourite option from either the question 1 or the question 2 in the following two questions:

Question 1. To choose between the following two options:

“A: Receiving \$467.00 on June 17th 2004”, and

“B: Receiving \$607.07 on June 17th 2005”.

Question 2. To choose between the following two options:

“C: Receiving 467.00 on June 16th 2005”, and

“D: Receiving 467.39 on June 17th 2005”.

Choosing delay (option B) in question 1 and no delay (option C) in question 2 is

inconsistent with the general hyperbolic discounting utility function $U(c_0, c_1, \dots, c_t) = v(c_0) + \sum_{t=1,2,\dots} \left(\prod_{s=1,2,\dots,t} \delta_s \right) v(c_t)$, where the function v is the concave instantaneous utility function, and $\delta_s < 1$ is a (weakly) increasing sequence therefore the time discounting factors in discrete time periods are not constant, namely $1, \delta_1, \delta_1 \delta_2, \dots, \prod_{s=1,2,\dots,t} \delta_s$.

Choosing “C” in question 2 implies that $\delta_{t^*} v(467.39) - v(467.39 - 0.39) < 0$, where $t^* = 17/06/2005$. The concavity of v and the fact that the discount rate is monotonic decreasing imply that $\delta_s v(x) - v(x - 0.39) < 0$ for any amount of money $x > 467.00$ and s earlier than June 17th 2005.

$$\begin{aligned} \delta_{t^*} v(607.07) &< v(607.07 - 0.39) \\ \delta_{t^*-1} \delta_{t^*} v(607.07) &< \delta_{t^*-1} v(607.07 - 2 \times (0.39)) < \delta_{t^*-365} v(607.07 - 2 \times (0.39)) \\ \delta_{t^*-2} \delta_{t^*-1} \delta_{t^*} v(607.07) &< \delta_{t^*-2} \delta_{t^*-365} v(607.07 - 3 \times (0.39)) < \delta_{t^*-366} \delta_{t^*-365} v(607.07 - 3 \times (0.39)) \\ &\vdots \\ \delta_1 \dots \delta_{t^*} v(607.07) &< \delta_1 \dots \delta_{t^*-365} v(607.07 - 365 \times (0.39)) \end{aligned}$$

Therefore, the straightforward calculation implies that $\left(\prod_{s=1,2,\dots,t^*} \delta_s \right) v(607.07) - \left(\prod_{s=1,2,\dots,t^*-365} \delta_s \right) v(607.07 - 365 \times (0.39)) < 0$. In fact that $607.07 - 365(0.39) < 140.07$ completes the proof. Therefore, choosing “B” in question 1 but choosing option “C” in question 2 are incompatible not only with the standard exponential discounted utility model but also with hyperbolic discounting theory.

Rubinstein (2003) reported that subjects would prefer B to A and C to D, which are obviously different from all generalised discounted utility model, because those models predict subjects would prefer either A & C or B&D or A&D but never B&C. Results of the experiment of Rubinstein (2003) is reported in Table 1.

Q 1	A	B	
	100(45%)	122(55%)	222
Q 2	C	D	
	126(54%)	108(46%)	234
	226	230	456

Table 1.

Two hundred forty-eight out of 456 subjects chose delay (option B) in question 1 and no delay (option C) in question2. If subjects' choices are made randomly, then such a result would be observed with a probability of 3%. The results demonstrate subjects' choice is inconsistent with the hyperbolic discounting theory.

Rubinstein (2003) thus argued that his experimental findings cannot be explained by the hyperbolic discounting theory. Instead, his findings may be better explained by the similarity procedure. The theoretical model of Rubinstein's similarity procedure is presented in Section 5.

4. Reference-Dependent Preferences

Apart from the diminishing discount rate, other anomalies mentioned in Section 2 could be understood as a misspecification of the instantaneous utility function. Based on findings from a number of psychological experiments, Kahneman and Tversky (1979) and Tversky and Kahneman (1991) propose three important modifications to instantaneous utility function – namely, that individuals' preferences are defined over changes of the level of wealth relative to his or her reference level, rather than depend merely on the absolute final level of wealth; that individuals are more sensitive to losses than gains; and that individuals have diminishing sensitivity for both gains and losses, captured by an S-shaped utility function that is concave over gains and convex over losses. In fact, an individual's utility is determined upon gains and losses instead of the final level of wealth was first analysed by Markowitz (1952), and is consistent with the way how people distinguish attributes such as brightness, loudness, and temperature comparing to previous levels, rather than evaluating those attributes in the term of absolute levels.

Kahneman and Tversky (1979) originally proposed the prospect theory³ for gambles with two non-zero outcomes. They proposed that when people face a gamble,

³ Tversky and Kahneman (1992) proposed a generalized vision of prospect theory, sometimes known as cumulative prospect theory, which extends the main result in the original prospect theory into gambles with more than two outcomes.

$(x, p; y, q)$, which means that the gamble offers an outcome x with probability p , and an outcome y with probability q , where $x \leq 0 \leq y$ or $y \leq 0 \leq x$, people evaluate the final outcome of this gamble as following:

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y),$$

where v is the value function, and π is the probability transformation function, presented in Diagram 1. In fact, the sum of probabilities $p + q$ may be less than unit, in this circumstance this gamble offers a zero outcome with a probability of $1 - p - q$.

Whenever $p + q = 1$ and either $x > y > 0$ or $x < y < 0$ the above utility function can be expressed as following:

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)].$$

Therefore, the utility or the value of a gamble with strictly positive outcomes or strictly negative outcomes can be represented as the value of outcome that is the small one in the term of absolute value plus the difference of value between the outcomes weighed by the transformed probability of the other one.

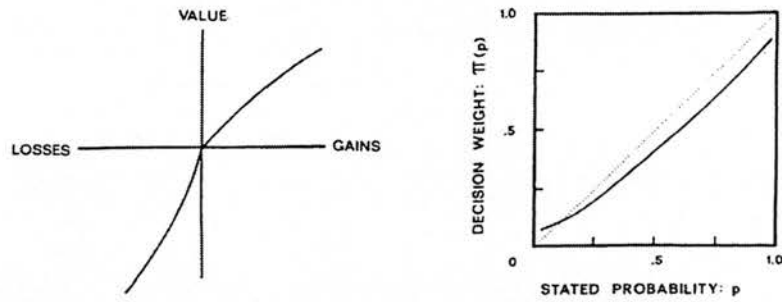


Diagram 1. Kahneman and Tversky's value function v and probability transformation function π

Kahneman and Tversky (1979) offered a series of experimental results, which violate the prediction of expected utility theory, as the evidences that people evaluate utility upon the changes of wealth status, gains and losses. The level of the reference points, which people calculate gains or losses relative to, are not constants, although in most circumstances the reference point coincides to the individual's current level of wealth. There are some other ways, in which people evaluate the gains and losses relative to a particular status of wealth either in the past or an expectation level of wealth that is different from the current one. Mellers, Schwartz, and Ritov (1999), and Breiter, Aharon, Kahneman, Dale, and Shizgal (2001) (see also Köszegi and Rabin (2006)) document that an individual's expectations of outcome significantly affect the determination of reference level. Therefore, the reference point may depend on the individual's memory of the past or expectation for the future.

Loewenstein and Prelec (1992) accommodate the S-shaped utility function with a separately defined discount function to explain anomalies that are inconsistent with predictions of the exponential discounted utility model, such as large outcomes are

discounted more than small outcomes; gains are discounted more than losses; and the delay-speedup asymmetry where people would require more compensations for delay an expected consumption than their willingness to pay to speed up an unexpected consumption.

In addition, Loewenstein and Adler (1995), Bowman, Minehart, and Rabin (1999), as well as Köszegi and Rabin (2006, 2007) proposed that an individual derives a consumption utility as well as a gain-loss utility from difference between the consumption relative and her reference level, where the gain-loss utility satisfies properties proposed by Kahneman and Tversky (1979) and Tversky and Kahneman (1991).

Bowman, Minehart, and Rabin (1999) explore a two-period consumption-saving model and predict that when information about an individual's future income is available, this information would significantly affect her consumption growth rate, which would be different from predictions of the standard Permanent Income Hypothesis⁴, that changes in future income should not affect the growth rate of consumption. The similar results are also found by Shea (1995 a, b).

⁴ By using panel data, Rizzo and Zeckhauser (2003) examined the effects of reference level of income on the behaviour of young male physicians. Rizzo and Zeckhauser (2003) reported that the reference level of income has a strong positive effect on subsequent income of those physicians whose current incomes are lower than their reference level, but has no effect on those physicians whose current incomes are equal to or higher than their reference level. Rizzo and Zeckhauser (2003) showed that the data is inconsistent with predictions of the traditional Permanent Income Hypothesis, but the data could be explained by the reference-dependent preferences.

Kőszegi and Rabin (2006) proposed that an individual's utility depends both on consumption, and on her reference level, and both of consumption and her reference level are K -dimensional vectors. She gets utility from consumption, which is additively separable across dimensions, and a gain-loss utility in each dimension. In each dimension, the gain-loss utility depends on a universal gain-loss function, which satisfies properties proposed by Kahneman and Tversky (1979), and Tversky and Kahneman (1991), captures utility caused by deviation of consumption level from her reference level.

In the work of Kőszegi and Rabin (2006), an individual's reference level is her current rational expectation about outcomes. Therefore, a person's expectation would significantly affect her preferences; on the other hand, her preferences would also significantly affect her expectation. In other words, her choices would depend on her reference level; at the meanwhile, her reference level would depend on her expected choices, which is capture by the concept of personal equilibrium.

A close work to the present thesis is done by Kőszegi and Rabin (2007), who develop a model of reference-dependent preferences, where an individual's utility depends both on consumption and recent changes in beliefs about present and future consumption, and she is loss-aversion over changes in planed consumption. In each time period, an individual's purpose is to maximize discounted expected utility given

her past expectations and her future behaviours and plans must be optimal. Köszegi and Rabin (2007) predict that an individual would prefer to receive all information together rather than separately to avoid her beliefs fluctuating; and she would also prefer to receive all information early rather than later because the earlier the information received the less the effect of the information on her beliefs and the less her beliefs fluctuating. Köszegi and Rabin (2007) also predict that in a circumstance of two-period consumption-savings an individual may over consume in early time period relative to her optimal plan and the more uncertainty about future the more over consumption is, but loss-aversion of lowering future consumption relative to her optimal plan will reduce her over consumption in early time period.

Apart from implications in analysing intertemporal decision-making, reference-dependent model⁵ has been widely used to analyse labour supply, anomalies observed in stock market study, etc. Camerer, Babcock, Loewenstein, and Thaler (1997) observe negative wage elasticities in NYC cab drivers based on the one-day unit of observation, which means that drivers' labour supply is driven by loss aversion, and their decision is made over a daily income target as their reference level.

Barberis, Huang, and Santos (2001) proposed a model of reference-dependent

⁵ Bateman, Munro, Rhodes, Starmer, and Sugden (1997) test a set of predictions derived from the theory of reference-dependent preferences, such as divergence between willingness-to-accept and willingness-to-pay, and conclude that the theory of reference-dependent preferences successfully explains these observations.

preferences, and explained equity premium puzzle, volatility puzzle, and predictability puzzle. The reference-dependent model illustrates a long period predictability in stock returns, which is consistent with the observed data. Therefore a positive dividend drives a high stock return, which in turn reduces the investor's risk aversion by the prior gains working as a cushion; furthermore the reduced risk aversion pushes the stock price upward further. A negative dividend, on the other hand, causes the investor to be more risk averse and pushes the stock price downward. Hence there is more volatility in stock returns than in the underlying dividends. In addition, the model generates a weak correlation between the stock returns and consumption. In the circumstance of heterogeneity, if the loss aversion is an aggregate characteristic of investors, and this loss aversion depends upon the prior stock market performances, the model still produces a high premium, high volatility, and predictability.

Heidhues and Köszegi (2005) develop a model where a profit-maximizing monopolist with uncertain cost of production sells to rational consumers with loss aversion. There are two circumstances of timing. In the first one, a monopolist, who sells to a single rational consumer with loss aversion, would first commit a pricing distribution, which the consumer observes. Then, the consumer would form expectations about her purchase behaviour. When the firm's price is realized, the consumer would suffer a shock to her willingness of purchase. After that, the

consumer would decide whether to buy. In the other situation, the consumer would first form expectations about her purchase behaviour. Then, the firm would observe its costs, commit a pricing distribution, and pin down a realized price. When the firm's price is realized, the consumer would suffer a shock to her willingness of purchase. After that, the consumer would decide whether to buy.

Heidhues and Köszegi (2005) predict that if the price is stochastically distributed, the consumer would suffer losses from purchase at higher price than what she expects or other realized prices, which is defined as comparison effect. This comparison effect, in turn, would be the force behind the price stickiness that means the firm would face a limited number of choices of price to charge even its marginal costs are continuously distributed, and the counter-cyclical markups, which means the firm would be reluctant to offer a low price when it takes into account that the consumer would compare the low price with any other higher price and decrease her willingness to buy when the price is high. Furthermore, Heidhues and Köszegi (2005) also predict that if the consumer expects purchase with a high probability, then her expectation would increase her willingness to buy. This probably explains why a firm would randomly offer low sale price to attract consumers' willingness to buy.

Heidhues and Köszegi (2006) develop a model of price competition with differentiated products, where consumer is loss averse over changes between her

recent expectations of purchase and the realized purchase price and her satisfaction with the realized purchase, and firms with uncertain costs of production decide on prices after observing their own realized cost of production. Heidhues and Köszegi (2006) find that consumer's sensitivity to loss aversion has a positive effect on the intensity of competition, such that the more consumers are loss averse the more demand is sensitive to price, which in turn leads to a higher intensive competition. In addition, this tendency is stronger at high price than at low price. Heidhues and Köszegi (2006) also predict that if and only if difference between two realized costs is not larger than a given constant, then there will be an equilibrium, where all firms will charge the same price for differentiated products. In addition, Heidhues and Köszegi (2006) predict that when firms face common stochastic costs, in any symmetric equilibrium, the countercyclical markup is decreasing in cost, and that in many situations prices are completely sticky. Furthermore, this tendency is stronger in less competitive industries.

Another close work to the present thesis is done by Tu (2004), who explores a structural model of reference-dependent preferences for intertemporal decision-making. Tu (2004) considered four scenarios, namely, delay of gains, delay of losses, speed-up of gains, and speed-up of losses, and estimated the coefficient of loss aversion in riskless intertemporal choice by using the panel data from a six-year Dutch representative household survey. In the survey, each subject would

face four questions, which would concern either delay or speed-up gains from winning a prize in the National Lottery or losses from paying a tax. In the time horizon of one year, Tu (2004) reported that the average coefficient of loss aversion is approximately two; the reference level of delay is larger than that of speedup; females are more sensitive to loss than males; and high-educated people and senior people are less loss-averse and more patient.

As one can see from above, alternative models of intertemporal decision-making mostly developed in two directions, either generalising the discounting function or incorporating more assumptions into the instantaneous utility function. These improvements in understanding time-preference benefit from importing more insights from psychological research. In the next section, I present the similarity procedure which is fundamentally different all generalized discounted utility models.

5. Rubinstein's Similarity Procedure

Most of the above-mentioned alternative models of intertemporal choice belong to a class of generalized discounted utility models as they employ two components – a time discounting function and an instantaneous utility function.

The similarity procedure, developed in Rubinstein (1988), is fundamentally different from the exponential discounted utility model and its alternatives mentioned in the previous sections as this procedure places the cognitive imperfections into the center of analysis. The concept of similarity was first introduced by Kahneman and Tversky (1977) and further defined by Rubinstein (1988).

In the process of decision-making under risk, Rubinstein (1988) emphasize that the decision maker would go through three steps following the similarity procedure. For a two-lotteries circumstance $L_1 = (x_1, p_1)$ and $L_2 = (x_2, p_2)$, people would first check if one lottery stochastically dominates the other. For instance, if both $x_1 > x_2$ and $p_1 > p_2$ then $L_1 \succ L_2$, the choice is done. If the first step is not decisive then move to step 2, and people would compare outcomes and the associated probability in terms of similarity or dissimilarity. For example, if $p_1 \sim p_2$, and $x_1 > x_2$ then $L_1 \succ L_2$, the option is selected. If the second step is not decisive, for example, the lotteries are similar on one dimension but dissimilar on the other, such as $x_1 \sim x_2$, and $p_1 > p_2$ then $L_1 \succ L_2$, the dissimilar dimension determine which option is selected. Otherwise, if above steps are not decisive then move to step 3, which is not specified.⁶

When an individual faces intertemporal choices, the probabilities of risky outcomes

⁶ For a similar theory (Vague theory) see Manzini and Mariotti (2004, and 2006).

are simply replaced by the time period in intertemporal choices. For example, conditional on that time is defined between 0 and 1, 0 in the intertemporal choices intuitively means a simultaneous event; on the other hand, 1 in the intertemporal choices intuitively means an event would happen in the infinite future.

The most problematic characteristic of this approach, as discussed in Rubinstein (1988) and in Rubinstein (2003), is that the third step might be inconsistent with the first two steps. Apart from the inconsistency between the first two steps and the third one, Rubinstein's similarity procedure explains experimental behaviour that could not be explained by either the standard exponential discounted utility model or hyperbolic discounting theory, but does not explain all the experimental behaviours, such as heterogeneity among subjects.

Leland (1994) modifies the similarity procedure proposed by Rubinstein (1988), and applies it to lotteries with multiple outcomes and probabilities, such as $L_1 = (x_{11}, p_{11}; x_{12}, p_{12}; \dots; x_{1n}, p_{1n})$ and $L_2 = (x_{21}, p_{21}; x_{22}, p_{22}; \dots; x_{2n}, p_{2n})$. In this situation, if a decision is not made in the first step, Leland (1994) emphasizes that the decision-maker would compare outcomes and their corresponding probability across lotteries in terms of equity or inequality to identify the dominant choice. Once each pair of comparisons is done, an option would be selected if it dominates the other in any pair of comparisons and is equal to the other in rest dimensions, such as

$x_{11} = x_{21}$ and $p_{11} = p_{21}$. Otherwise, the decision-maker would process the third step, in which she would compare outcomes and their corresponding probability across lotteries in terms of similarity and dissimilarity, to identify the optimal choice. If there is still not a decision made, then choice would be randomly selected.

In addition, Leland (2000) applies the similarity-based procedure in analysing examples of intertemporal choices to offer explanation of anomalies of the exponential discounted utility model, such as common ratio, common difference, and reflection effect, and conclude that if choice is decided upon, at least in part, similarity procedure, those anomalies could be reasonably understood by the similarity procedure.

6. The Relationship of the Present Thesis to the Existing Literature

An increasing number of researchers both in the field of economics and psychology pointed out that some of the fundamental assumptions in the traditional economics are unrealistic in the terms of psychology, and attempted to offer some alternative models which may improve our understanding of human behaviour. Surprisingly,

there have been little work done that apply insights from psychological research to one of the most important human activity, intertemporal decision-making.

Among many alternative models developed in response to recent experimental evidence, reference-dependent model emerged as one of the more realistic and more capable of capturing individual choices in the static situation. Thus, my main contribution to the existing literature is to apply the reference-dependent model to intertemporal decision making and to show that one can model intertemporal choices without the assumptions of time discounting.

In the Chapter II, I present an alternative to the exponential discounted utility model and the hyperbolic discounting theory. My alternative approach is based on the reference-dependent preferences, which is described in Section 4 of Chapter I. The closest work to the novel theoretical model presented in Chapter II is Köszegi and Rabin (2007). In addition, my novel approach (presented in Chapter II) could also offer an alternative explanation of observations by Rubinstein (2003) described in Section 3.2 of Chapter I.

In Chapter III, I further extend the model of Chapter II to a situation where an individual is uncertain about her future reference point. In Chapter III, an individual's reference levels are assumed to follow a random walk process to capture

uncertainty about future circumstances. Hence, her reference levels could increase, decrease, or remain the same relatively to her current reference level. The intertemporal reference-dependent model may offer an alternative explanation for the present bias, future bias, and dynamic inconsistency over time discussed in Section 2.2 of Chapter I.

I find that the novel reference-dependent model of intertemporal choice, presented in Chapters II and III, could explain some of the anomalies of intertemporal choice – namely, the future bias, and the dynamic inconsistency over time – even without the assumption of time discounting.

Finally, in Chapter IV, I present the results of a survey-based within-subject choice experiment and explore whether subjects' behaviour is consistent with three possible theories of intertemporal choice – namely, the hyperbolic discounting theory (described in Section 2 of Chapter I), Rubinstein's similarity procedure (presented in Section 5 of Chapter I), and the novel intertemporal model of reference-dependent preferences, (explored in Chapter II and Chapter III of the present thesis). While I find that my novel theoretical model does not explain all subjects' behaviour, it nevertheless performs no worse than Rubinstein's similarity procedure, and better than the hyperbolic discounting theory.

Intertemporal Reference-Dependent Model

Abstract: In this chapter, I extend the reference-dependent model to intertemporal choices and show that one can model intertemporal choice even without the assumption of time discounting. Using the idea of mental accounting, I assume that in each time period, the individual cares about the absolute level of consumption in that time period, and, in addition, about how this level of consumption compares to the reference consumption in that time period. Moreover, when the consumption level falls short of her reference level, she experiences loss aversion. Thus, her instantaneous utility has two components, and I assume that these two components are additively separable. While there are multiple ways of how individual reference points could be formed, in this chapter I only focus on three simple ways of reference point determination – choice-set dependent, choice-set independent, and hybrid reference points. In this chapter, I also argue that the intertemporal reference-dependent model may offer an alternative explanation of the experimental observations by Rubinstein (2003).

1. Introduction

The growing body of psychological experimental findings suggests that individuals' preferences are significantly different from what is commonly assumed in neo-classical economic theory. As one of the most important human activities, intertemporal decision-making involves tradeoffs among consumptions in different time periods. For at least one hundred years, economists have been trying to understand and explain how individuals make decisions over intertemporal perspectives, with literature on the subject dating back as early as to Adam Smith.

Since Samuelson (1937) developed his parsimonious model of intertemporal preferences by compressing all psychological factors underlying intertemporal choices into a single parameter, the exponential discounted utility model has dominated economic analysis of intertemporal choices. Although the exponential discounted utility model has been applied to analyse a large number of intertemporal choice situations, including savings behaviour, labour supply, etc., it has also received much criticism from both empirical research on intertemporal choices and experimental results in cognitive psychology (see Frederick, Loewenstein, and O'Donoghue (2002), also see Section 2.2 of Chapter I of this thesis). In response to those criticisms, economists have proposed a number of alternatives by either

modifying the assumption of constant discount rate or incorporating more assumptions of the instantaneous utility function.

Probably the most prominent alternative to the exponential discounted utility model is the hyperbolic discounting theory, which was first introduced by Strotz (1956) and further studied by Phelps and Pollak (1968). This theory is based on the findings from studies of animal and human behaviour in cognitive psychology. Especially after Laibson (1997) applied hyperbolic discounting theory to explore the role of illiquid assets, such as housing, as an imperfect technology of self-control, hyperbolic discounting theory become more and more influential. Hyperbolic discounting theory concerns the consistency of an individual's time preferences. It captures the phenomenon that a person's time preferences exhibit a relatively high discount rate over short time horizon and a relatively low discount rate over long time horizon. In other words, time discount rate is declining over time rather than constant over time. In addition, the hyperbolic discounting theory also implies the present bias in a sense that a person would choose to realize consumption immediately rather than delaying them for even for a little bit.

Although hyperbolic discounting theory successfully modified the assumption of constant discount rates to capture the phenomenon of diminishing time discount rates, its descriptive validity has been challenged by a recent experimental study, which is

presented in Section 3.2 of Chapter I, by Rubinstein (2003), who suggested that in the experiments designed to challenge the standard neoclassical theory of intertemporal choice, the similarity procedure⁷ would fit experimental data better than hyperbolic discounting theory. When faced with two intertemporal options, an individual would compare outcomes and the associated date in terms of similarity and dissimilarity, and choose the favourite one. As discussed in Rubinstein (1988) as well as in Rubinstein (2003), if decision is not made in two specified decisive steps then similarity procedure does not have specified predictions, which may be inconsistent with the previous steps.

In this chapter, I extend the reference-dependent model based on the idea of loss aversion studied due to Kahneman and Tversky (1979), presented in Section 4 of Chapter I⁸, and the idea of mental accounting due to Thaler (1980, 1985, 1999), in the formulation developed by Köszegi and Rabin (2006), where an individual's instantaneous utility depends not only upon her current consumption level but also on her current reference level, to intertemporal choices. Specially, an individual's instantaneous utility is composed of an intrinsic consumption utility and a reference-dependent gain-loss utility for each time period. Furthermore, both consumption and the reference level are additively separable over time periods in the

⁷ The concept of similarity was first introduced by Tversky (1977) and further defined by Rubinstein (1988).

⁸ Since the influential work of Kahneman and Tversky (1979), the idea of reference-dependent preferences has been applied in analysis of intertemporal decision-making, for example, see Loewenstein and Prelec (1992), Bowman, Minehart, and Rabin (1999) and Köszegi and Rabin (2007).

problem of intertemporal decision-making. In contrast to the existing models of intertemporal choice, there is no time discounting and no return on saving.

While there are multiple ways of how individual reference points could be formed, in this paper I only focus on three simple ways of reference point determination. Here, I consider a case where, as it is common in the literature on reference-dependent preferences, an individual's reference is either choice-set dependent or, alternatively, choice-set independent. I also consider a hybrid reference level whereby some components of the reference point can be choice-set dependent and others can be choice-set independent. In this chapter, I also apply the intertemporal reference-dependent model to explain experimental data of Rubinstein (2003), and argue that the intertemporal reference-dependent model may offer an alternative explanation of experimental observations by Rubinstein (2003) for some range of parameters of the novel model.

I thus show that one can model intertemporal choice even without the assumption of time discounting. Furthermore, in Chapter III of the present thesis I explore a more complicated process of reference point determination, and show that a number of intertemporal choice anomalies can be explained by the intertemporal reference-dependent model.

This chapter is organized as follows: Section 2 describes the intertemporal reference-dependent model, introduces choice-set dependent and choice-set independent reference points, and presents a procedure of intertemporal decision-making. Section 3 presents an application of this model to the experimental setup of Rubinstein (2003). Section 4 considers formation of hybrid reference points. Summary and further directions are presented in Section 5.

2. Reference-Dependent Preferences

The intertemporal reference-dependent model presented here employs reference points and loss aversion, which are two important concepts in behavioural economics. Kahneman and Tversky (1979) proposed that an individual's preferences are defined over changes of wealth relative to her reference level, rather than depend merely on the absolute level of final wealth; and individuals are more sensitive to losses than gains; in addition, individuals reveal diminishing sensitivity for both gains and losses, which gives an S-shaped utility function that is concave over domain of gains and convex over domain of losses.

2.1. Intertemporal Reference-Dependent Model

The model presented here is based on the linear formulation used in Köszegi and Rabin (2006). Here, an individual's utility in each time period depends not only upon the consumption of an item but also on the deviation between consumption and her reference level that is her anticipation of outcomes. Importantly, here, there is no interest rate on savings and individuals value time equally (i.e. no time discounting).

Specifically, a person receives an intrinsic consumption utility $m(c_t)$, and a reference-dependent gain-loss utility $g(c_t|r_t)$ in each time period, in which c_t is her per-period consumption level and r_t is her per-period reference level. Time is discrete, $t \in 1, 2, \dots, T$, and an individual's per-period utility could be written as:

$$u(c_t, r_t) = m(c_t) + g(c_t|r_t), \quad t \in 1, 2, \dots, T.$$

In each time period, the gain-loss utility function depends upon the difference between the utility of consumption and the utility of reference point, $m(c_t) - m(r_t)$. Therefore the gain-loss utility could be specified as $g(c_t|r_t) = \mu(m(c_t) - m(r_t))$, where the function μ captures loss aversion, and satisfies the following properties originally proposed by Kahneman and Tversky (1979) and further specified by Bowman, Minehart, and Rabin (1999), which include a kink at zero:

A0. $\mu(x)$ is continuous for all x , and twice differentiable for $x \neq 0$, and $\mu(0) = 0$.

A1. $\mu(x)$ is strictly increasing in x .

A2. If $y > x > 0$, then $\mu(y) + \mu(-y) < \mu(x) + \mu(-x)$.

A3. $\mu''(x) \leq 0$ for $x > 0$ and $\mu''(x) \geq 0$ for $x < 0$.

A4. $\frac{\mu'_+(0)}{\mu'_-(0)} = \lambda > 1$, where $\mu'_+(0) = \lim_{x \rightarrow 0} \mu'(|x|)$ and $\mu'_-(0) = \lim_{x \rightarrow 0} \mu'(-|x|)$.

Property A0 and A1 imply that $u(c_t, r_t)$ is increasing in c_t and decreasing in r_t .

Property A2 and A4 capture the concept of loss aversion, in a sense that losses are more important than gains, which imply that individuals are risk-averse in the domain of gains but risk-seeking in the domain of losses.

In addition, the consumption utility function $m(x)$ satisfies the following properties:

B0. $m(x)$ is continuous for all x , and twice differentiable for $x > 0$, $m(0) = 0$.

B1. $m(x)$ is strictly increasing.

B2. $m''(x) < 0$ for all x .

Furthermore, both the intrinsic consumption utility and the reference-dependent gain-loss utility are separable additive over time dimension⁹, such that

⁹ Both consumption and gain-loss could be defined over hedonic dimensions. For example, the consumption utility from watching a movie would be composed of utility from the following four hedonic dimensions: acting, plot, cinematography, and music. In this context, two goods are perfect substitutes if they are hedonically similar to an individual. Here I extend the standard reference-dependent model, and assume that both consumptions and reference levels are K-dimensional consumption sequences in each time period; hence consumption $c_t = (c_{t1}, c_{t2}, \dots, c_{tK}) \in \mathbb{R}^K$ and reference level $r_t = (r_{t1}, r_{t2}, \dots, r_{tK}) \in \mathbb{R}^K$. Therefore, a given consumption bundle (reference level) provides k-dimensional hedonic value in each period of time, which could be represented by a $K \times T$ matrix. For instance, a consumption bundle of

$m(c) = \sum_{t=1}^T m(c_t)$, where c_t is the consumption at time period of t , and $g(c|r) = \sum_{t=1}^T \mu(m(c_t) - m(r_t))$, where r_t is the reference level at time period of t .

Therefore an individual's lifetime utility is represented by

$$U(c, r) = \sum_{t=1}^T m(c_t) + \sum_{t=1}^T \mu(m(c_t) - m(r_t)).$$

Thus, an individual would prefer $L_a = (x_a, t_a)$ to $L_b = (x_b, t_b)$ in a situation with two intertemporal options, for example, if and only if $U(x_a, r_a) > U(x_b, r_b)$.

2.2. Choice-set Dependent vs. Choice-set Independent Reference Points

The reference points are important component in reference-dependent model. As Kahneman and Tversky (1979) point out, the reference levels are not necessarily constant. Although in most circumstances the reference point coincides with the individual's current level of wealth, the reference level might also be either a particular past consumption level or a future expected level of wealth that is different from the current one. Mellers, Schwartz, and Ritov (1999), and Breiter, Aharon, Kahneman, Dale, and Shizgal (2001) (see also Köszegi and Rabin (2006)) document that an individual's expectations of outcome are significantly more important than

watching four movies in four time periods would be represented by the following matrix: $\begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{pmatrix}$, in which each column presents a movie with four hedonic consumption values in each time period, and each row of the matrix presents one hedonic consumption dimension, namely, acting, plot, cinematography, and music. Specially, a monetary consumption bundle provides a one-dimensional hedonic value at each period of time.

previous consumption status on determination of reference level. For example, an individual would be less happy with a 5% increase in her income when she expected a 10% one, on the other hand, she would experience more happiness with a 15% raise in income compared to her expectation of a 10% increase.

In the intertemporal setting, time preferences might be affected by current expectation of future outcomes. Because individual's reference points are very difficult to observe and estimate, there are no clear evidences of reference point formation in the literature on reference-dependent preferences. Based on the idea of mental accounting – that according to different “mental account” individuals may treat monetary expenditures as different types – studied by Thaler (1980, 1985, and 1999), to fix the reference point, we make following assumptions:

Assumption C1: The reference point is the consumption bundle which offers the highest consumption value at the earliest available date.

For example, apply different references to each pair of intertemporal options, and in a situation with two pairs of monetary intertemporal options, and each option involve four time periods, $A:(a_1, a_2, a_3, a_4)$ and $B:(b_1, b_2, b_3, b_4)$, $C:(c_1, c_2, c_3, c_4)$ and $D:(d_1, d_2, d_3, d_4)$, in which $a_1 > b_1 > c_1 > d_1$. Assumption C1 implies that if an individual faces a choice between a pair of intertemporal options A and B, her

reference level is determined by option $A:(a_1, a_2, a_3, a_4)$ as this option that offers higher consumption at the earliest date, but when faced with the pair of intertemporal options C and C, her reference level is $C:(c_1, c_2, c_3, c_4)$.

Assumption C2: The reference point is choice-set independent, with the reference level of a monetary consumption bundle is $X:(x_1, x_2, \dots, x_t, \dots)$, for $x_t \geq 0$ $t \in 1, 2, \dots, T$.

For the above-mentioned example, Assumption C2 implies that same reference point will be applied to both pairs of intertemporal options, because the reference level is choice-set independent as $X:(x_1, x_2, x_3, x_4)$. I also consider the following special case of Assumption C2.

Assumption C2': The reference point is choice-set independent, and involves an immediate payoff in today and zero payoff thereafter, e.g. the reference of a monetary consumption bundle is, $X:(x, 0, \dots, 0, \dots)$, for $x \geq 0$.

In the above-mentioned example, Assumption C2' implies that the reference level is choice-set independent as $X:(x, 0, 0, 0)$ for both pairs of intertemporal options and is a special case of Assumption C2. The Assumption C2' may capture the reference points of subjects in economic experiments, as they are typically recruited with a

promise of an immediate payment (and nothing afterwards). For example, if subjects were recruited using an advertisement that stating that “on average, subjects earn £25”, their reference point may be (£25,0,0,0).

In the next sub-section, examples would be offered to explain how the reference-dependent model works in intertemporal circumstances. To simplify analyses and without losing generality, a linear modification of the intertemporal reference-dependent model with Assumption C1 and C2' will be applied in example 1 in the next sub-section.

2.3. A Procedure of Intertemporal Decision-Making

In this sub-section, the procedure of intertemporal decision-making would be analysed. Let $A:(a_1, a_2, a_3, a_4)$ and $B:(b_1, b_2, b_3, b_4)$ be a pair of monetary intertemporal options, in which $a_1 > b_1$, and \succ and \sim denote strict preference and indifference, respectively. Assumption C1 implies that the reference point for this pair of intertemporal options is $A:(a_1, a_2, a_3, a_4)$, hence an individual who follows Assumption C1 would have following time preferences:

$$A \succ \sim \prec B \text{ iff } U(A|A) = \sum_{t=1}^4 m(a_t) \succ = < U(B|A) = \sum_{t=1}^4 m(b_t) + \sum_{t=1}^4 g(b_t|a_t).$$

On the other hand, Assumption C2 implies that the reference point for this pair of intertemporal options is $X:(x_1, x_2, x_3, x_4)$, hence an individual who follows Assumption C2 would have following time preferences:

$$A \succsim B \text{ iff } U(A|X) = \sum_{t=1}^4 m(a_t) + \sum_{t=1}^4 g(a_t|x_t) \succsim U(B|X) = \sum_{t=1}^4 m(b_t) + \sum_{t=1}^4 g(b_t|x_t).$$

To simplify further analysis, assume that $m(x_t) = x_t$, $\mu(x_t) = \eta x_t$, $\mu(-x_t) = -\lambda \eta x_t$, where $x_t > 0$, $\eta > 0$, and $\lambda > 1$. In addition, assume the function μ is identical for each time period, hence $\lambda_t \equiv \lambda$ and $\eta_t \equiv \eta$.

Example 1: Let two pairs of intertemporal options be $A:(0, a, 0, 0)$ and $B:(0, 0, 0, b)$, $C:(0, 0, c, 0)$ and $D:(0, 0, 0, d)$, in which $0 < a < c < d < b$. Assume a group of individuals are required to choose the favourite option from either the pair of options A and B or the pair of options C and D.

If an individual follows the Assumption C1, in the first pair of options the reference level is $A:(0, a, 0, 0)$, and then the utility of option A is given by

$$U_A(c, r) = U_A((0, a, 0, 0)|(0, a, 0, 0)) = m(a) = a,$$

and the utility of option B is given by

$$\begin{aligned} U_B(c, r) &= U_B((0, 0, 0, b)|(0, a, 0, 0)) = m(b) + g(0|a) + g(b|0) \\ &= b - \lambda \eta a + \eta b \end{aligned}$$

In addition, applying Assumption C1 in the second pair of options implies that the reference level is $C : (0,0,c,0)$ and the utility of option C is given by

$$U_C(c,r) = U_C((0,0,c,0)|(0,0,c,0)) = m(c) = c,$$

and the utility of option D is given by

$$\begin{aligned} U_D(c,r) &= U_D((0,0,0,d)|(0,0,c,0)) = m(d) + g(0|c) + g(d|0) \\ &= d - \lambda\eta c + \eta d \end{aligned}$$

		1 st pair	2 nd pair
$\frac{d}{c} \geq \lambda$	$\eta > 0$	$A \prec B$	$C \prec D$
	$\eta > 0$	$A \prec B$	$C \prec D$
	$\eta > 0$	$A \prec B$	$C \prec D$
$\frac{b}{a} \geq \lambda > \frac{d}{c}$	$\eta > \frac{b-a}{\lambda a - b}$	$A \prec B$	$C \succ D$
	$\frac{d-c}{\lambda c - d} \leq \eta < \frac{b-a}{\lambda a - b}$	$A \prec B$	$C \sim D$
	$\eta < \frac{d-c}{\lambda c - d}$	$A \prec B$	$C \prec D$
$\lambda > \frac{b}{a}$	$\eta \geq \frac{b-a}{\lambda a - b}$	$A \succ B$	$C \succ D$
	$\frac{d-c}{\lambda c - d} < \eta < \frac{b-a}{\lambda a - b}$	$A \prec B$	$C \succ D$
	$\eta < \frac{d-c}{\lambda c - d}$	$A \prec B$	$C \prec D$

Table 2.

Hence, the individual would prefer A to B if and only if $\eta > \frac{b-a}{\lambda a - b} > 0$, prefer B to A if and only if $\frac{b-a}{\lambda a - b} > \eta > 0$, and be indifferent between A and B if and only if $\eta = \frac{b-a}{\lambda a - b} > 0$, where $\lambda > \frac{b}{a} > 1$. On the other hand, the individual would always

prefer B to A if $\frac{b}{a} > \lambda > 1$ for all range of $\eta > 0$. Similarly, the individual would prefer C to D if and only if $\eta > \frac{d-c}{\lambda c-d} > 0$, prefer D to C if and only if $\frac{d-c}{\lambda c-d} > \eta > 0$, and be indifferent between C and D if and only if $\eta = \frac{d-c}{\lambda c-d} > 0$, where $\lambda > \frac{d}{c} > 1$. On the other hand, the individual would always prefer D to C if $\frac{d}{c} > \lambda > 1$ for all range of $\eta > 0$. Following Assumption C1, her preferences over these two intertemporal options are summarized in the Table 2 according to different rang of parameters of the gain-loss utility.

Thus, following the Assumption C1, the individual's time preferences when presented with a choice over intertemporal options $A:(0,a,0,0)$ and $B:(0,0,0,b)$, will be different from her time preferences when presented with options $C:(0,0,c,0)$ and $D:(0,0,0,d)$.

On the other hand, if the individual follows the Assumption C2', then the reference point is same for all intertemporal options, which is fixed at $X:(x,0,...,0,...)$, in which $x \geq 0$. According to the Assumption C2' the utility of option A in the first pair of choices is given by

$$\begin{aligned} U_A(c,r) &= U_A((0,a,0,0)|(x,0,0,0)) = m(a) + g(0|x) + g(a|0) \\ &= a - \lambda\eta x + \eta a \end{aligned}$$

and the utility of option B is given by

$$\begin{aligned} U_B(c,r) &= U_B((0,0,0,b)|(x,0,0,0)) = m(b) + g(0|x) + g(b|0) \\ &= b - \lambda\eta x + \eta b \end{aligned}$$

In addition, applying Assumption C2' in the second pair of intertemporal options implies that the utility of option C is given by

$$\begin{aligned} U_C(c, r) &= U_C((0, 0, c, 0) | (x, 0, 0, 0)) = m(c) + g(0|x) + g(c|0) \\ &= c - \lambda \eta x + \eta c, \end{aligned}$$

and the utility of option D is given by

$$\begin{aligned} U_D(d, r) &= U_D((0, 0, 0, d) | (x, 0, 0, 0)) = m(d) + g(0|x) + g(d|0) \\ &= d - \lambda \eta x + \eta d \end{aligned}$$

Since all four options involve a zero payoff today, which is captured by the first number of each option, an individual would suffer losses on the earliest date in each option. Therefore, she would always prefer B to A as long as $b > a$, and would also always prefer D to C as long as $d > c$, for $\eta > 0$ and $\lambda > 1$.

In the example 1, when facing intertemporal choices, a group of individuals with different rules of reference point determination would have different time preferences. In addition, if individuals' reference levels are choice-set dependent, the intertemporal reference-dependent model would also predict heterogeneous time preferences among individuals that would vary according to the parameters of the gain-loss utility. Based on the analysis of the above-mentioned example, the intertemporal reference-dependent model may offer an alternative explanation of the experimental observation of Rubinstein (2003), which will be presented in the next section.

3. Application of Reference-Dependent Model to Rubinstein (2003)

Over the last two decades, a number of alternatives of the exponential discounted utility model have been developed by generalizing assumptions of either the time discounting function or the instantaneous utility function, (for example, hyperbolic discounting theory that successfully describes the diminishing discount rates). In a recent study, Rubinstein (2003) (see Section 3.2 of Chapter I of this thesis) introduced experimental observations, which challenge descriptive validity of not only the standard exponential discounted utility model but the hyperbolic discounting theory with an increasing and concave instantaneous utility function as well.

For example, in the study of Rubinstein (2003), subjects were asked (in the year of 2002) to choose the favourite option from either the question 1 or the question 2 in the following two questions:

Question 1.

"A: Receiving \$467.00 on June 17th 2004." $\Rightarrow A:(0,467,0,0)$

"B: Receiving \$607.07 on June 17th 2005." $\Rightarrow B:(0,0,0,607.07)$

Question 2.

"C: Receiving 467.00 on June 16th 2005." $\Rightarrow C:(0,0,467,0)$

"D: Receiving 467.39 on June 17th 2005." $\Rightarrow D:(0,0,0,467.39)$

In each of above brackets, the first number presents the monetary payoff on today, the second number presents the monetary payoff on June 17th 2004, and third number presents the monetary payoff on June 16th 2005, and the final number presents the monetary payoff on June 17th 2005. Experimental findings reveal that subjects tend to prefer B to A but C to D. This is inconsistent with prediction of not only the standard exponential discounted utility model but also the hyperbolic discounting theory with an increasing and concave instantaneous utility function, since the standard exponential discounted utility model would predict that an individual may either prefer A to B and C to D or prefer B to A and D to C, and the hyperbolic discounting theory with an increasing and concave instantaneous utility function would predict that she may also prefer A to B and D to C, but both models predict that she would never prefer B to A and C to D at the same time (for a more complete analysis see Section 3.2 of Chapter I).

Is the similarity approach the only explanation of subjects' behaviour in the Rubinstein's experiment? I suggest that the intertemporal reference-dependent model

may offer an alternative explanation of observations reported by Rubinstein (2003)¹⁰.

Following Assumption C1, in the first pair of options an individual's reference would be $A:(0,467,0,0)$, therefore receiving \$607.07 on 17th 2005 would involve a loss on 17th 2004 but a significant gain on 17th 2005. On the other hand, in the second pair of options her reference would be $C:(0,0,467,0)$, therefore receiving \$467.39 on 17th 2005 would involve a loss on 16th 2005 but a small gain on 17th 2005.

Following Assumption C2', an individual's reference level would be choice-set independently fixed at $X:(x,0,0,0)$, hence both options involve a loss on today but receiving \$607.07 on 17th 2005 would involve a significant gain on 17th 2005, which is more than receiving \$467 on 17th 2004 in the first question; in the second question, both options involve a loss on today but receiving \$467.39 on 17th 2005 involve more gains than receiving \$467 on 16th 2005.

Therefore, the experiment of Rubinstein (2003) may be understood using the example 1 of Section 2. Following the Assumption C1, individuals' time preferences associated with different range of parameter λ and η are summarized in Table 3, in which $a = c = \$467$, $b = \$607.07$, and $d = \$467.39$. Following the Assumption C1, in some range of parameter λ and η , the intertemporal reference-dependent

¹⁰ Rubinstein's experimental results can also be explained by the fact that people tend to have difficulties of computing results on a one-year investment (Question 1) and a one-day investment (Question 2) – I thank Joe Swierzbinski for pointing this to me.

model would predict that an individual would prefer option B in the first question but option C in the second question, which may imply that loss aversion would force subjects to require a significant compensation for delaying any consumption, which is consistent with prediction of similarity procedure.

On the other hand, if we follow the Assumption C2', the intertemporal reference-dependent model would predict that an individual would always prefer B in the first question and D in the second pair of intertemporal options, which is consistent with one of possible predictions of both neoclassical and hyperbolic discounting theory.

		Q1	Q2
$\frac{d}{c} \geq \lambda$	$\eta > 0$	$A \prec B$	$C \prec D$
	$\eta > 0$	$A \prec B$	$C \prec D$
	$\eta > 0$	$A \prec B$	$C \prec D$
$\frac{b}{a} \geq \lambda > \frac{d}{c}$	$\eta > \frac{b-a}{\lambda a-b}$	$A \prec B$	$C \succ D$
	$\frac{d-c}{\lambda c-d} \leq \eta < \frac{b-a}{\lambda a-b}$	$A \prec B$	$C \succ D$
	$\eta < \frac{d-c}{\lambda c-d}$	$A \prec B$	$C \prec D$
$\lambda > \frac{b}{a}$	$\eta \geq \frac{b-a}{\lambda a-b}$	$A \succ B$	$C \succ D$
	$\frac{d-c}{\lambda c-d} < \eta < \frac{b-a}{\lambda a-b}$	$A \prec B$	$C \succ D$
	$\eta < \frac{d-c}{\lambda c-d}$	$A \prec B$	$C \prec D$

Table 3. Parameter range consistent with Assumption C1, here $\lambda > 1$, $a = c = 467$, $b = 607.07$, and

In summary, the intertemporal reference-dependent model may offer an alternative explanation of observations by Rubinstein (2003), and may also offer an explanation of the heterogeneity among subjects.

4. Hybrid Reference Points

In Section 2, two kinds of rule of determining reference level are presented, namely choice-set dependent level and choice-set independent reference level. In addition, in Section 2, an example is offered to explain how these two kinds of reference level may work. Apart from the choice-set dependent and choice-set independent reference levels, individual may follow a hybrid rule of determining reference level, e.g. mixing the choice-set dependent and choice-set independent reference levels up. For example, in the example 1, the choice-set dependent reference level of the first pair of intertemporal options is $A:(0,a,0,0)$ and the choice-set independent reference level is fixed at $X:(x,0,0,0)$. If the individual forms a “maximum hybrid” expectation, the hybrid reference level is $H_{Max}:(x,a,0,0)$. If an individual follows maximum rule of hybrid reference level, she would tend to experience losses.

For example, following the above hybrid assumption, the utility of option A in

example 1 would be given as:

$$U_A(c, r) = U_A((0, a, 0, 0)(x, a, 0, 0)) = m(a) + g(0|x) \\ = a - \lambda \eta x$$

and the utility of option B is given by

$$U_B(c, r) = U_B((0, 0, 0, b)(x, a, 0, 0)) = m(b) + g(0|x) + g(0|a) + g(b|0) \\ = b - \lambda \eta x - \lambda \eta a + \eta b$$

Therefore, the individual would prefer A to B if and only if $\eta > \frac{b-a}{\lambda a - b} > 0$, prefer B to A if and only if $\frac{b-a}{\lambda a - b} > \eta > 0$, and be indifferent between A and B if and only if $\eta = \frac{b-a}{\lambda a - b} > 0$, where $\lambda > \frac{b}{a} > 1$. On the other hand, the individual would always prefer B to A if $\frac{b}{a} > \lambda > 1$ for all range of $\eta > 0$. Similarly, the individual would prefer C to D if and only if $\eta > \frac{d-c}{\lambda c - d} > 0$, prefer D to C if and only if $\frac{d-c}{\lambda c - d} > \eta > 0$, and be indifferent between C and D if and only if $\eta = \frac{d-c}{\lambda c - d} > 0$, where $\lambda > \frac{d}{c} > 1$. On the other hand, the individual would always prefer D to C if $\frac{d}{c} > \lambda > 1$ for all range of $\eta > 0$. This result is same as the prediction of applying the Assumption C1 in Section 2.

The more generalized rule of maximum hybrid reference level is summarized in the following assumption:

Assumption H1: In each time period, in the case of two intertemporal options A and B, the reference points would be $r_{kt} = \max(a_{kt}, b_{kt}, x_{kt})$, where a_{kt} is an element of the intertemporal option A, b_{kt} is an element of the intertemporal option B, and x_{kt}

is an element of choice-set independent reference level.

With this type of reference points, subjects would tend to experience losses. Here, optimistic expectations lead to lower welfare.

On the other hand, the individual may follow the rule of “minimum hybrid” reference levels, which is given in the following assumption:

Assumption H2: In each time period, in the case of two intertemporal options A and B, the reference points would be $r_{kt} = \min(a_{kt}, b_{kt}, x_{kt})$, where a_{kt} is an element of the intertemporal option A, b_{kt} is an element of the intertemporal option B, and x_{kt} is an element of choice-set independent reference level.

With this type of reference points, subjects would tend to avoid experience of losses. Here pessimistic expectations lead to higher welfare.

In the example 1, according to the Assumption C2', the choice-set independent reference level is fixed at $X : (x, 0, 0, 0)$, hence the hybrid reference level would be

$H_{Min} : (0, 0, 0, 0)$. Then utility of option A would be given as:

$$\begin{aligned} U_A(c, r) &= U_A((0, a, 0, 0) | (0, 0, 0, 0)) = m(a) + g(a | 0) \\ &= a + \eta a \end{aligned}$$

and the utility of option B is given by

$$\begin{aligned} U_B(c, r) &= U_B((0, 0, 0, b) | (0, 0, 0, 0)) = m(b) + g(b | 0) \\ &= b + \eta b \end{aligned}$$

If an individual follows the special rule of “minimum hybrid” reference points in the example 1, she would always prefer option B to A as long as $b > a$, for $\eta > 0$ and $\lambda > 1$, which is the same as the prediction of applying choice-set independent reference points Assumption C2’ in Section 2, except she would experience more gains. Following the rule of “minimum hybrid” reference points, the individual would always prefer D to C as long as $d > c$, for $\eta > 0$ and $\lambda > 1$, which is same as the prediction of applying choice-set independent reference points Assumption C2’ in Section 2.

5. Summary and Further Directions

This chapter suggests an alternative approach of how individuals make choices over time. The proposed model is based on the idea of loss aversion suggested by Kahneman and Tversky (1979), and the idea of mental accounting suggested by Thaler (1980, 1985, and 1999). Here, an individual’s utility is composed of an intrinsic consumption utility and a gain-loss utility in each time period. In this model, individual’s choices depend on individuals’ expectation of how much money she may earn. I show that the reference-dependent model can be used to explain intertemporal

choices even without the assumption of time discounting. In addition, the proposed intertemporal reference-dependent model has been applied to experimental observations of Rubinstein (2003). Here, if the individual's reference point is choice-set dependent by the bundle which offers highest payoff at the earliest date, for some parameters of individual's gain-loss utility, the predictions are consistent with Rubinstein's (2003) experimental observations. If, instead, individual's reference point is choice-set independent, the prediction coincides with one of possible predictions of hyperbolic discounting theory.

Since there is little evidence of how subjects form their reference points in each time period, we are agnostic about the true reference-generating process. While Köszegi and Rabin (2007) propose a model via reference points are determined in what they call a "personal equilibrium", further research on the reference generating process is needed.

Can Loss Aversion Explain Intertemporal Choice Anomalies?¹¹

Abstract: In this chapter, I extend the intertemporal model of reference-dependent preferences of Chapter II, where individual is uncertain about her future circumstances. Here, an individual's instantaneous utility is composed of an intrinsic consumption utility and a reference-dependent gain-loss utility, and her reference levels follow a random walk process. While the model does not explicitly include time discounting and return on saving, it nevertheless may offer an alternative explanation of present bias and negative time preference (future bias). In addition, the intertemporal reference-dependent model may also offer an alternative explanation of dynamically inconsistent preferences over time.

¹¹ I would like to thank József Sákovics for useful and constructive suggestions.

1. Introduction

Intertemporal decision-making is important and ubiquitous human activity. To explain how people make choices over time, economists have been using the exponential discounted utility model of Samuelson (1937), where an individual's preferences between two consumption prospects should merely depend upon the absolute time interval that separates them. However, in the past few decades, this model has received substantial criticism, and as the result, a large number of alternative theories of intertemporal choice have been developed by economists (see Frederick, Loewenstein, and O'Donoghue (2002)).

In this chapter, I extend the intertemporal model of reference-dependent preferences of Chapter II, in which an individual's instantaneous utility is composed of an intrinsic consumption utility and a reference-dependent gain-loss utility for each time period to a situation where individual's reference levels follow a random walk process to capture uncertainty about future circumstances. Furthermore, both consumption and the reference level are additively separable over time periods in the problem of intertemporal decision-making. In contrast to the existing models of intertemporal choice, there is no time discounting and no return on saving.

There are many reasons why an individual may face uncertainty about her future circumstance, and three classes of reasons are worthy of attention. The first class of reasons is psychological – for example, individual's reference point may depend on her emotional state or mood. Alternatively, a change in the reference point may represent a change in the individual's taste. The second class of reasons is economic. Suppose, for example, that an individual's reference point is simply a standard of living that this individual aspires to achieve. The expenditure required to achieve this standard of living may depend on the state of the economy (e.g. on whether the economy is expanding or contracting). The third class of reasons is social. For example, suppose, again, an individual aspires to achieve some standard of living, but this standard of living may depend on what do other people consume, or on consumption fashions, or on some other social factors.

As one of the major results, with no time discounting and no return on saving, the intertemporal model of reference-dependent preferences may offer an alternative explanation of present bias and negative time preference (future bias). Another major result in this chapter is that with no time discounting and no return on saving, the intertemporal reference-dependent model may also offer an alternative explanation of dynamically inconsistent preferences over time.

Hyperbolic discounting model, first studied by Strotz (1956) and further advanced by

Laibson (1997), is the most influential alternative to the neoclassical theory of intertemporal decision-making. Hyperbolic discounting theory provides a general mathematical expression of psychology for intertemporal decision-making, and captures that an individual's preferences are dynamically inconsistent over time. For example, hyperbolic discounting theory predicts that an individual would prefer to receive \$100 right now rather than to receive \$110 tomorrow, but at the same time, she would also prefer to receive \$110 in one year and one day from now rather than to receive \$100 in one year from now.

Experimental evidence has not only revealed that an individual's intertemporal preferences are inconsistent over time but also that in some circumstances an individual would prefer to delay a given amount of consumption. In other words, individuals would have negative time preference (i.e. future bias), which could not be explained by either the standard exponential discounted utility model or the hyperbolic discounting theory (see Koopmans et al (1964), Olson and Bailey (1981), Loewenstein and Prelec (1991), and Loewenstein and Sicherman (1991)).

Reference-dependent preferences were first introduced by Kahneman and Tversky (1979), in which an individual's preferences are defined over changes of wealth relative to her reference level; individuals care more about losses than gains; individuals reveal diminishing sensitivity for both gains and losses, which means that

an S-shaped utility function that is concave over the domain of gains and convex over the domain of losses with a kink at zero. Since the influential work of Kahneman and Tversky (1979), the model of reference-dependent preferences has been applied in analysis of intertemporal decision-making, for example, see Loewenstein and Prelec (1992), Bowman, Minehart, and Rabin (1999) and Köszegi and Rabin (2007).

This chapter is organized as follows: Section 2 describes the intertemporal reference-dependent model, and the random walk process of reference level that captures the uncertainty about future circumstances; Section 3 is devoted to discussion of an individual's preferences for delaying an indivisible consumption value over time, in which an individual would have present bias or future bias; Section 4 describes how her intertemporal preferences are dynamically inconsistent over time, where her preferences over two different consumption values would change when both are delayed by one time period; further direction and conclusion are given in Section 5.

2. Reference-Dependent Preferences

Kahneman and Tversky (1979) proposed that an individual's preferences are defined over changes of wealth relative to her reference level, rather than depend merely on the absolute level of final wealth; and individuals care more about losses than gains; in addition, individuals reveal diminishing sensitivity for both gains and losses, which means that an S-shaped utility function that is concave over the domain of gains and convex over the domain of losses with a kink at zero. In other words, individuals are risk-averse in gains but risk-loving in losses.

In this section, I develop a reference-dependent model of intertemporal decision-making, in which an individual's per-period utility is composed of an intrinsic consumption utility and a reference-dependent gain-loss utility (in the formulation appearing in Köszegi and Rabin (2006)), and where an individual's reference levels follow a stochastic process, which captures uncertainty about future circumstance.

2.1 Loss Aversion

Suppose there is no interest rate on savings and individuals value time equally (i.e. no time discounting). An individual's per-period utility is composed of an intrinsic consumption utility $m(c_t)$ and a utility from the reference-dependent gain-loss

sensation $g(c_t|r_t)$ that measures the effect of difference between consumption and her reference level. Therefore her per-period utility could be represented by a function

$$u(c_t, r_t) = m(c_t) + g(c_t|r_t),$$

where c_t is her consumption level and r_t is her reference level in each time periods, $t \in 1, 2, \dots, T$.

In the per-period utility function, the gain-loss utility depends upon the difference between the utility of consumption and the utility of the reference point, $m(c_t) - m(r_t)$. Specifically, the gain-loss utility could be represented by

$$g(c_t|r_t) = \mu(m(c_t) - m(r_t)),$$

where the function μ , which is identical for each time period, captures loss aversion, and satisfies properties, which are originally proposed by Kahneman and Tversky (1979) and further specified by Bowman, Minehart, and Rabin (1999), as following:

A0. $\mu(x)$ is continuous for all x , and twice differentiable for $x \neq 0$, and $\mu(0) = 0$.

A1. $\mu(x)$ is strictly increasing in x .

A2. If $y > x > 0$, then $\mu(y) + \mu(-y) < \mu(x) + \mu(-x)$.

A3. $\mu''(x) \leq 0$ for $x > 0$ and $\mu''(x) \geq 0$ for $x < 0$.

A4. $\frac{\mu'_+(0)}{\mu'_-(0)} = \lambda > 1$, where $\mu'_+(0) = \lim_{x \rightarrow 0} \mu'(|x|)$ and $\mu'_-(0) = \lim_{x \rightarrow 0} \mu'(-|x|)$.

Property A0 and A1 imply that $u(c_t, r_t)$ is increasing in c_t and decreasing in r_t .

Property A2 and A4 capture the concept of loss aversion, in other words, losses are more important than gains, which imply that individuals are risk-averse in the domain of gains but risk-seeking in the domain of losses. The intrinsic consumption utility function $m(c)$ is a monotonic increasing concave function as described by neo-classical economic theory.

Furthermore, both the intrinsic consumption utility and the reference-dependent gain-loss utility are separable additive over T ¹². Hence, the overall utility of a consumption bundle could be written as following function:

$$U(c, r) = \sum_{t=1}^T m(c_t) + \sum_{t=1}^T \mu(m(c_t) - m(r_t)). \quad (1)$$

For simplicity, throughout this chapter, we will consider the following linear formulation:

$$\left. \begin{array}{l} \text{Intrinsic consumption utility } m(x_t) = x_t, \\ \text{Gains } \mu(x_t) = \eta x_t, \text{ for } x_t > 0, \eta > 0, \text{ and } \lambda > 1 \\ \text{Losses } \mu(-x_t) = -\lambda \eta x_t, \text{ for } x_t > 0, \eta > 0, \text{ and } \lambda > 1. \end{array} \right\} \quad (2)$$

¹² Both consumption and reference levels could be defined over \mathfrak{R}^K . In other words, a given consumption bundle provides K-dimensional hedonic value in each time period, such as colour, taste, etc. A monetary measured consumption bundle merely provides one-dimensional hedonic value in each time period.

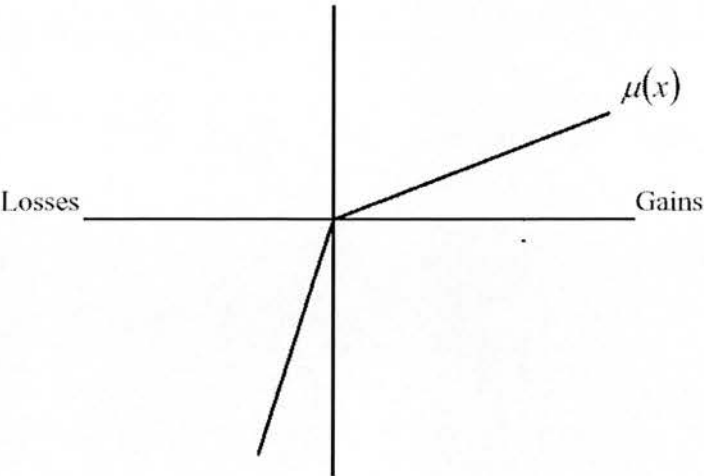


Diagram 2. The linear formulation of gain-loss utility $\mu(x)$

Notice that the function μ is identical for each time period, hence $\lambda_t \equiv \lambda$ and $\eta_t \equiv \eta$. In other words, an individual has constant sensitivity about losses over time.

Here, as an individual exhibits loss aversion, delaying a given amount of consumption value would involve losses in the earlier time periods. On the other hand, postponing consumption would be acceptable if it provides enough gains to compensate earlier losses.

2.2 Changing Reference Points

Ryder and Heal (1973) proposed that an individual's current reference points would

change with her previous consumption and reference point in a dynamic utility-maximization framework. Specifically, an individual's preference points would change with the function $r_t = \alpha c_{t-1} + (1 - \alpha)r_{t-1}$, where $\alpha \in (0,1)$ measure the speed at which the reference points adjust in response to her most recent consumption. Based on the work of Ryder and Heal (1973), Bowman, Minehart, and Rabin (1999) apply the reference changing function in a two-period consumption-saving model and reveal that when an individual's consumption level captures uncertainty about future incomes, her responses to uncertainty by increasing or decreasing her current consumption level are different.

Mellers, et al (1999), and Breiter, et al (2001) (see also Köszegi and Rabin (2006)) document that an individual's reference level is significantly dependent on her expectations of future outcomes. Hence, when an individual faces an intertemporal choice, her reference level would also be responsive to uncertainty about future circumstances. For example, information about our future health circumstances may not change our expected incomes, but may affect our current consumption decision.

In an intertemporal setting, an individual's current choices may be affected by her all possible realized future reference points, which represent uncertainty of future circumstance. By recognizing this, the individual may incorporate effects of randomly realized reference points into her current decision-making. To simplify the

analysis of effects of randomly realized reference points on an individual's current decision-making, in this chapter, there are only three possible reference levels in the next time period conditional on her current reference level and all information about future circumstances are received in the current time period and imminently captured in the stochastic process. Thus the individual's future reference levels would increase, decrease, or remain in the same level as the previous period. This reference generating process could either be an internal process – for example, an individual's anticipations, and thus her reference level, may be optimistic or pessimistic, depending on her mood upon waking up in the morning. Alternatively, her reference level is the expenditure required to purchase certain basket of goods, and it thus may change because the prices of goods may change.

Let r_{t-1} be the reference level in the time period of $t-1$. Therefore, an individual's reference level in the time period of t could be represented by following probability functions:

$$\left. \begin{aligned} \text{prob}(r_t = r_{t-1} - \alpha) &= p, \\ \text{prob}(r_t = r_{t-1}) &= 1 - p - q, \\ \text{prob}(r_t = r_{t-1} + \alpha) &= q, \end{aligned} \right\} (3)$$

where $\alpha > 0$, which is the magnitude of reference level reduced or increased per-period. In addition, the initial reference point r_1 is exogenously determined. In other word, an individual's reference is generated by an initial reference and

anticipations about future. Probabilities capture the uncertainty of future circumstances. The random walk process over three time periods is represented in Diagram 3.

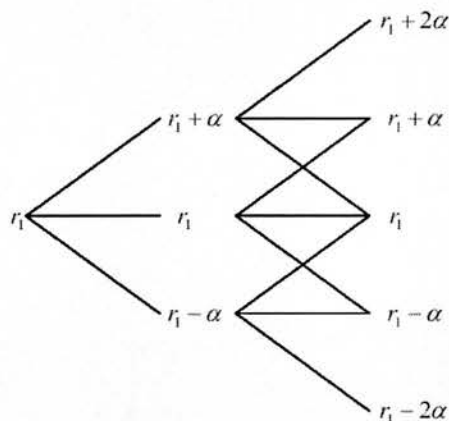


Diagram 3. The random walk process over three time periods, where $\text{prob}(r_t = r_{t-1} - \alpha) = p$, $\text{prob}(r_t = r_{t-1}) = 1 - p - q$, and $\text{prob}(r_t = r_{t-1} + \alpha) = q$.

For example, the anticipations of future circumstances may lead the individual's reference level to increase, decrease or remain at the previous level with equal probability in the second time period. In other words, $p = 1 - p - q = q = \frac{1}{3}$, where $\text{prob}(r_2 = r_1 - \alpha) = p$, $\text{prob}(r_2 = r_1) = 1 - p - q$, and $\text{prob}(r_2 = r_1 + \alpha) = q$. An individual, whose objective is to maximize her expected lifetime utility, faces two intertemporal options: $A: (c, 0)$, and option $B: (0, c)$, where $c = r_1$. With the linear formulation previously described, utilities from two options are as following:

The utility from option A:

$$\begin{aligned}
 E[U_A(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\
 &= m(c) + g(c|r_1) + \frac{1}{3}(m(0) + g(0|r_1 - \alpha)) + \frac{1}{3}(m(0) + g(0|r_1)) + \frac{1}{3}(m(0) + g(0|r_1 + \alpha)) \\
 &= c - \frac{1}{3}\lambda\eta r_1 - \frac{1}{3}\lambda\eta(r_1 + \alpha) - \frac{1}{3}\lambda\eta(r_1 - \alpha) \\
 &= c - \lambda\eta r_1
 \end{aligned}$$

The utility from option B:

$$\begin{aligned}
 E[U_B(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\
 &= m(0) + g(0|r_1) + \frac{1}{3}(m(c) + g(c|r_1 - \alpha)) + \frac{1}{3}(m(c) + g(c|r_1)) + \frac{1}{3}(m(c) + g(c|r_1 + \alpha)) \\
 &= -\lambda\eta r_1 + \frac{1}{3}c + \frac{1}{3}(c - \lambda\eta\alpha) + \frac{1}{3}(c + \eta\alpha) \\
 &= c - \lambda\eta r_1 - \frac{1}{3}\lambda\eta\alpha + \frac{1}{3}\eta\alpha
 \end{aligned}$$

Since $\lambda > 1$, therefore $E[U_A] - E[U_B] = \frac{1}{3}\lambda\eta\alpha - \frac{1}{3}\eta\alpha > 0$, the utility from option A is higher than that from option B. An individual who maximizes her expected lifetime utility would prefer option A to option B. In other words, when uncertainty about tomorrow involves equal probability of increasing or reducing her reference levels, she would strictly prefer to enjoy consumption today rather than delaying it for tomorrow. Intuitively, giving up the guaranteed indivisible monetary consumption today would incur a large amount of losses today, which could not be compensated by receiving the guaranteed indivisible monetary consumption value tomorrow even if her reference level tomorrow would probably be lower than that of today.

3. Intertemporal Preferences over Constant Consumption

In this section, I will apply the above intertemporal reference-dependent model with stochastic reference points to explore an individual's intertemporal choice. Importantly, I assume here that there is no interest rate and no time discounting.

3.1 Two Period Model

We first consider an individual's intertemporal decisions over two time periods when she faces an amount of guaranteed indivisible monetary consumption value in either time period. Therefore, she could either consume this indivisible consumption value in the first time period (today hereafter) and nothing in the second time period (tomorrow thereafter), or vice versa. In the intertemporal context, these two intertemporal options could be represented as: option $A: (c, 0)$, and option $B: (0, c)$.

Any uncertainty is resolved in the second period caused by news or information about tomorrow, which would affect the individual's anticipations of tomorrow, and directly affects her reference levels of tomorrow. Hence, there are three possible reference levels of tomorrow, $\text{prob}(r_2 = r_1 - \alpha) = p$, $\text{prob}(r_2 = r_1) = 1 - p - q$, and

$prob(r_2 = r_1 + \alpha) = q$, where $\alpha > 0$ that represents the magnitude of reference level decreased or increased by news about future. In addition, since consumption could not be negative in either period, it is reasonable to assume that the reference levels in the second time period would not be negative, hence $r_1 - \alpha \geq 0$.

An individual's purpose is to maximize her expected lifetime utility, and her expected lifetime utility is given by:

$$E[U(r_1, c_1, r_2, c_2)] = m(c_1) + g(c_1|r_1) + E\{m(c_2) + g(c_2|r_2)\},$$

where c_t is her consumption in each period and r_t is her reference level in each period, $t \in (1, 2)$.

Let us consider the earlier example, where the individual's reference level would increase, decrease or remain as the previous level with equal probability in the second time period. The utility from option A:

$$\begin{aligned} E[U_A(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\ &= m(c) + g(c|r_1) + \frac{1}{3}(m(0) + g(0|r_1 - \alpha)) + \frac{1}{3}(m(0) + g(0|r_1)) + \frac{1}{3}(m(0) + g(0|r_1 + \alpha)) \\ &= c - \frac{1}{3}\lambda\eta r_1 - \frac{1}{3}\lambda\eta(r_1 + \alpha) - \frac{1}{3}\lambda\eta(r_1 - \alpha) \\ &= c - \lambda\eta r_1 \end{aligned}$$

The utility from option B:

$$\begin{aligned}
 E[U_B(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\
 &= m(0) + g(0|r_1) + \frac{1}{3}(m(c) + g(c|r_1 - \alpha)) + \frac{1}{3}(m(c) + g(c|r_1)) + \frac{1}{3}(m(c) + g(c|r_1 + \alpha)) \\
 &= -\lambda\eta r_1 + \frac{1}{3}c + \frac{1}{3}(c - \lambda\eta\alpha) + \frac{1}{3}(c + \eta\alpha) \\
 &= c - \lambda\eta r_1 - \frac{1}{3}\lambda\eta\alpha + \frac{1}{3}\eta\alpha
 \end{aligned}$$

The gains from option A is zero since the consumption level is equal to her initial reference level, and the gains from option B is $\frac{1}{3}\eta\alpha$ that comes from difference between consumption level and her possible pessimistic reference level $r_1 - \alpha$, hence option B provides more gains than option A. At the same time, losses from option A is $-\lambda\eta r_1$, and losses from option B is $-\lambda\eta r_1 - \frac{1}{3}\lambda\eta\alpha$, in which $-\frac{1}{3}\lambda\eta\alpha$ comes from difference between consumption level and her optimistic reference level $r_1 + \alpha$, hence option B also provides more losses than option A. If the individual's reference level in the second time period increases, decreases or remains at the previous-period level with equal probability, the individual would prefer option A to option B since the amount by which gains outweigh losses is higher with option A rather than with option B. Hence the individual would maximize her expected lifetime utility by choosing the intertemporal option, which, in fact, minimizes her expected lifetime combination of gains and losses in terms of absolute value.

We now turn to the more general case when reference levels may increase, decrease or stay unchanged with unequal probabilities. All calculations for two time periods can be found in Appendix A. Here, only a summary is presented. Diagram 4

represents the relative position of the consumption value to her possible reference level of tomorrow, where c^I , c^{II} , c^{III} , and c^{IV} are four possible level of the consumption value; and $r_1 - \alpha$, r_1 , and $r_1 + \alpha$ are three possible reference level of tomorrow, along with the probability p , $1 - p - q$, and q respectively.

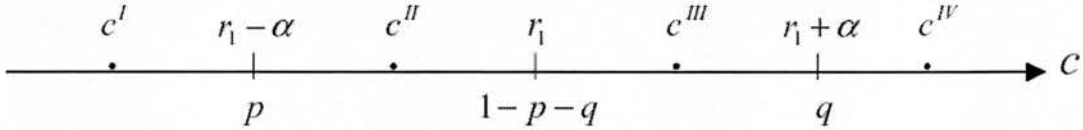


Diagram 4. The range of consumption value for two-period model, where $\text{prob}(r_t = r_{t-1} - \alpha) = p$,

$$\text{prob}(r_t = r_{t-1}) = 1 - p - q, \text{ and } \text{prob}(r_t = r_{t-1} + \alpha) = q.$$

When the consumption value is $c = c^I$, where $\frac{c - r_1}{\alpha} \leq -1$, the consumption value is lower than the individual's pessimistic reference level of tomorrow, in which only losses are involved in her expected lifetime utility. In other words, both options provide zero gains, and $E[U_A] = E[U_B]$ for all parameters of probability. Therefore, an individual would be indifferent between two intertemporal options. Since the consumption value is strictly lower than her pessimistic reference level, from either the view of today or the view of tomorrow, two intertemporal options would involve zero gains and incur large amount of losses, which are same in the term of expected lifetime utility.

When the consumption value is $c = c^{II}$, where $-1 < \frac{c - r_1}{\alpha} \leq 0$, the consumption value is between her reference level of today and her pessimistic reference level of

tomorrow. The individual would maximize her expected lifetime utility by choosing the intertemporal option, which minimizes her expected lifetime combination of gains and losses. Therefore, the option A would provide lower combination of gains and losses in terms of absolute value than option B as long as her expected reference level of tomorrow is possibly lower than her reference level of today. In other words, the inequality $E[U_A] - E[U_B] = p(c - r_1 + \alpha)(\lambda - 1)\eta > 0$ holds as long as $p > 0$. In addition, the closer the consumption value to her reference level of today, the lower the probability of reducing reference level is when the individual prefer option A over option B. Therefore, the individual, whose objective is to maximize her expected lifetime utility, would always strictly prefer any given consumption value today rather than delaying that for tomorrow if her reference level of tomorrow would possibly decrease. Since her references depend on her anticipations about tomorrow, from the view of tomorrow, if the realized circumstance is either at the level of $r_1 + \alpha$ or at the level of r_1 , both intertemporal options incur same amount of expected gains. Therefore, in these two circumstances, an individual would be indifferent between these two intertemporal options. On the other hand, when a possible pessimistic anticipation of tomorrow is involved, her expected utility would be maximized by choosing option A, which would provide lower combination of gains and losses in terms of absolute value than option B. Hence, when an individual has pessimistic expectations about tomorrow with nonzero probability, she would always prefer consumption today to tomorrow, so that the present bias arises.

When the consumption value is $c = c^m$, where $0 < \frac{c - r_1}{\alpha} \leq 1$, the consumption value is between her reference level of today and her optimistic reference level of tomorrow. The individual's expected lifetime utility would be maximized by choosing the option A, which minimize her expected combination of gains and losses in terms of absolute value as long as the ratio of the probability of reducing her reference level to the probability of increasing her reference level is high enough. In other words, the inequality $E[U_A] - E[U_B] = p(\lambda - 1)\eta\alpha - q(\lambda - 1)\eta(c - r_1) > 0$ holds as long as $\frac{p}{q} > \frac{c - r_1}{\alpha}$. When the consumption value is $c = c^m$, an individual's preference over the two intertemporal options depends upon anticipations about the future, which would increase or decrease her reference level of tomorrow. When there is a higher ratio of the probability of reducing her reference level to the probability of increasing her reference level, the individual would prefer to choose the consumption today. Furthermore, the closer the consumption value to her optimistic reference level of tomorrow, the higher the ratio is when the individual prefers option A over option B. Intuitively, if her pessimistic reference level realizes tomorrow, the amount of gains from consumption in a pessimistic circumstance is higher than that in an optimistic circumstance, but still not enough to cover losses from today. On the hand, when there is more probability to have a higher reference, the individual would prefer to delay the consumption, since not to do so would incur more losses tomorrow than that of today; to reduce possible losses, she would give

up gains from today and delay the consumption to tomorrow. Therefore, the individual would prefer consumption today to consumption tomorrow when the ratio of her pessimistic reference level of tomorrow to her optimistic reference level of tomorrow is high enough.

When the consumption value is $c = c^{IV}$, where $1 < \frac{c - r_1}{\alpha}$, the consumption value is higher than her optimistic reference level tomorrow. The individual's expected lifetime utility would be maximized by choosing the option A, which minimize her expected combination of gains and losses in terms of absolute value as long as the probability of reducing her reference level is higher than the probability of increasing her reference level. In other words, the inequality $E[U_A] - E[U_B] = (p - q)(\lambda - 1)\eta\alpha > 0$ holds as long as $p > q$. Intuitively, when the possibility of decreasing her reference is higher, delaying consumption to tomorrow would involve more gains as well as losses than consumption day, hence she would prefer consumption today to consumption tomorrow.

Therefore, under the circumstance of two time periods, her preferences over these two intertemporal options as discussed above are summarized in Proposition 1.

Proposition 1. Suppose an individual's intertemporal utility function is given by (1) and (2) where reference points are determined by (3). Suppose further that the

individual faces a choice between option A: $(c,0)$, and option B: $(0,c)$. Then, depending on the combination of the fixed consumption value and initial reference level and parameters of the random walk, an individual may have future bias, present bias, or may be indifference between two options. In particular,

(i). (Future Bias) An individual will have future bias when the fixed consumption value exceeds her current reference level and her reference level has more chance to increase rather than decrease in next time period.

(ii). (Indifference) An individual will be indifference between two options when the fixed consumption value falls short of her pessimistic reference level of next time period.

(iii). (Present Bias) An individual will have present bias when the fixed consumption value falls short of her current reference level but still exceeds her pessimistic reference level of next time period and her reference level has more chance to decrease rather than increase in next time period.

One can find more complete text of the proposition in the Appendix A.

Notice that both the future bias and the indifference between two options could not be explained by either exponential discounted utility model (see Section 2.1 of Chapter I) or the hyperbolic discounting theory (see Section 3.1 of Chapter I). In fact, future bias and indifference are considered to be anomalies of the intertemporal

choice (see Section 2.2 of Chapter I). The diagram 5 presents areas of present bias, future bias, and indifferent preferences; the vertical axis measures the difference between the probability of increasing her reference levels and the probability of decreasing her reference levels; and the horizontal axis measures consumption levels.

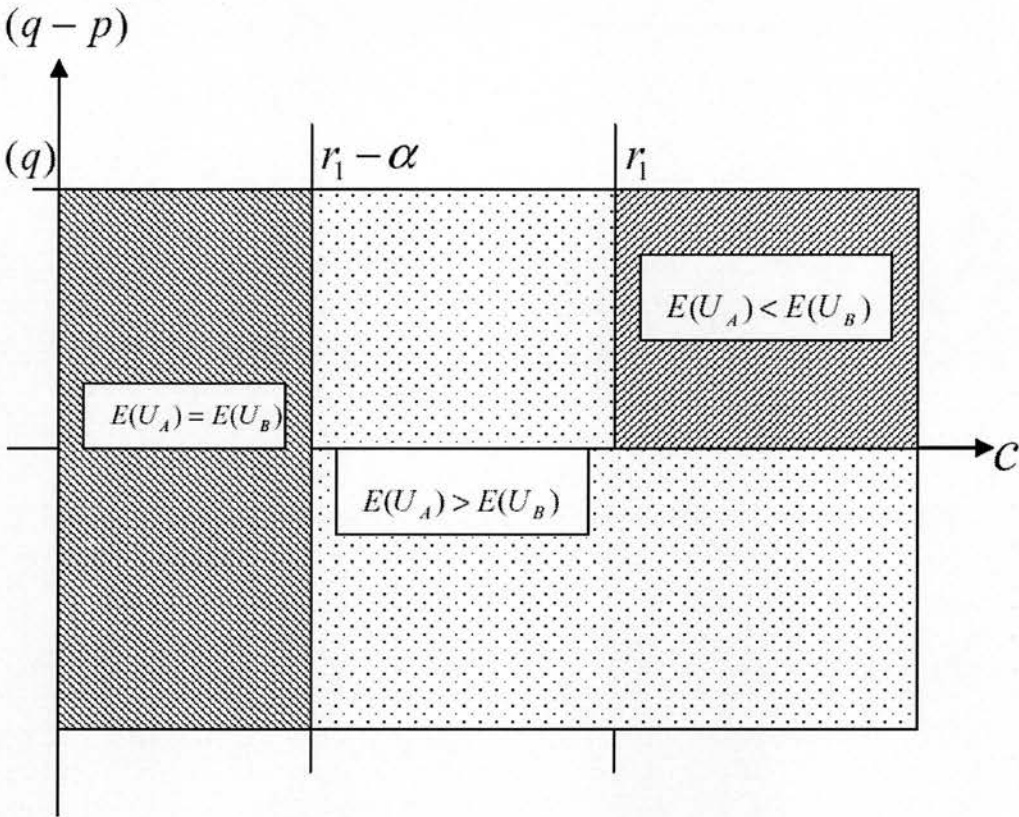


Diagram 5. Areas of present bias, future bias, and indifferent preferences

3.2 Three Periods Model

Let us now consider a three periods model, in which the consumption value could be

consumed in exactly one of three time periods, where the individual would choose the consumption value today but nothing tomorrow and nothing the day after tomorrow; or other wise choose consumption either tomorrow or the day after tomorrow but nothing for other two days. In the intertemporal context, the individual would choose among the following three intertemporal options: option $A: (c, 0, 0)$, option $B: (0, c, 0)$, and $C: (0, 0, c)$.

Following the random walk process, there are three possible reference levels of tomorrow, with probabilities $\text{prob}(r_2 = r_1 - \alpha) = p$, $\text{prob}(r_2 = r_1) = 1 - p - q$, and $\text{prob}(r_2 = r_1 + \alpha) = q$; and there are five possible reference levels of the day after tomorrow, with probabilities $\text{prob}(r_3 = r_1 - 2\alpha) = p^2$, $\text{prob}(r_3 = r_1 - \alpha) = 2p(1 - p - q)$, $\text{prob}(r_3 = r_1) = 2pq + (1 - p - q)^2$, $\text{prob}(r_3 = r_1 + \alpha) = 2q(1 - p - q)$, $\text{prob}(r_3 = r_1 + 2\alpha) = q^2$, where $\alpha > 0$ is the magnitude by which the reference level has reduced or increased by news about future in the random walk process.

Suppose an individual maximizes her expected lifetime utility, which is given by:

$$E[U(r_1, c_1, r_2, c_2)] = m(c_1) + g(c_1 | r_1) + E\{m(c_2) + g(c_2 | r_2) + m(c_3) + g(c_3 | r_3)\},$$

where c_t is her consumption in each period, and r_t is her reference level in each period, $t \in (1, 2, 3)$.

The most interesting outcome in the three-period model is that an individual's lifetime expected utilities would be decreasing if the given amount of consumption were delayed period by period, as well as the marginal utility of her lifetime utilities. The calculations of results can be found in Appendix B. Here, only a summary is presented.

Proposition 2. Suppose an individual's intertemporal utility function is given by (1) and (2) where reference points are determined by (3). Suppose further that the individual faces a choice between option A: $(c,0,0)$, option B: $(0,c,0)$, and option C: $(0,0,c)$. Then, depending on the combination of the fixed consumption value and initial reference level and parameters of the random walk, an individual may have constant marginal utility over time, increasing marginal utility over time, and diminishing marginal utility over time over three options.

One can find more complete text of the proposition in the Appendix B

Notice that both the constant marginal utility over time and the increasing marginal utility over time could not be explained by either exponential discounted utility model (see Section 2.1 of Chapter I) or the hyperbolic discounting theory (see Section 3.1 of Chapter I). In fact that, the standard exponential discounted utility theory and the hyperbolic discounting theory only predict that the individual will

have a diminishing marginal utility (see Section 2.1 and Section 3.1 of Chapter I).

4. Intertemporal Trade-off and Dynamically Inconsistent Preferences

In the previous section we consider the case when consumption value was constant over time. Now we will look into the situation when the individual faces a trade-off between smaller but earlier consumption and larger but later one. As one can see, loss aversion may lead to a possibility of dynamically inconsistent preferences.

4.1 Dynamically Inconsistent Preferences

Consider a situation when an individual is faced with a choice over two intertemporal options, one involving a small and sooner payoff and the other involving a large but later payoff. According to the exponential discounted utility model, an individual's choice should depend merely on the absolute time interval separating the two options, but independent of the date of each intertemporal option. However, experimental studies showed that an individual's preferences over these two intertemporal options might change when both of them are delayed by a given time periods. For example,

in a study of Thaler (1981), an individual might prefer one apple today to two apples tomorrow, but at the same time she might prefer two apples in 51 days to one apple in 50 days.

Consider the following options, a choice between a consumption value c today but nothing for tomorrow and the day after tomorrow, a consumption value $c + \beta$ tomorrow but nothing for today and the day after tomorrow, that is $A: (c, 0, 0)$ and $B: (0, c + \beta, 0)$, where $\beta > 0$; at the same time, a choice between the consumption value c tomorrow but nothing for today and the day after tomorrow, and the other consumption value $c + \beta$ the day after tomorrow but nothing for today and tomorrow, that is $C: (0, c, 0)$ and $D: (0, 0, c + \beta)$, where $\beta > 0$. The intertemporal reference-dependent model predicts that the individual would have dynamically inconsistent preference over these two consumption values when both are delayed by one time period, such that she would prefer option A to option B, but prefer option D to option C at the same time.

Based on the random walk generating process, there are three possible reference levels in the second period, with probabilities, $\text{prob}(r_2 = r_1 - \alpha) = p$, $\text{prob}(r_2 = r_1) = 1 - p - q$, and $\text{prob}(r_2 = r_1 + \alpha) = q$, where $\alpha > 0$ represents the magnitude by which reference level is reduced or increased by news about the future in the random walk process. Furthermore, there are five possible reference levels in

the third period, with probabilities, $prob(r_3 = r_1 - 2\alpha) = p^2$,
 $prob(r_3 = r_1 - \alpha) = 2p(1 - p - q)$, $prob(r_3 = r_1) = 2pq + (1 - p - q)^2$,
 $prob(r_3 = r_1 + \alpha) = 2q(1 - p - q)$, $prob(r_3 = r_1 + 2\alpha) = q^2$.

The individual's expected lifetime utility is given by:

$$E[U(r_1, c_1, r_2, c_2)] = m(c_1) + g(c_1 | r_1) + E\{m(c_2) + g(c_2 | r_2) + m(c_3) + g(c_3 | r_3)\},$$

where c_t is her consumption in each period, and r_t is her reference level in each period, $t \in (1, 2, 3)$. Furthermore, the individual's expected utilities of a given amount of consumption value and differences between two consecutive expected utilities are decreasing over time when the consumption value is in the neighbourhood of her reference level of today, therefore let $-1 < \frac{c - r_1}{\alpha} < \frac{c + \beta - r_1}{\alpha} \leq 1$. The general calculations of results can be found in Appendix C. Here, only a summary is presented.

Proposition 3. Suppose an individual's intertemporal utility function is given by (1) and (2) where reference points are determined by (3). Suppose further that the individual faces two set of choices: a choice between A: $(c, 0, 0)$ and B: $(0, c + \beta, 0)$, and a choice between C: $(0, c, 0)$ and D: $(0, 0, c + \beta)$, where $\beta > 0$. Then an individual will have dynamically inconsistent preferences over two consumption values (c and $c + \beta$) when both are delayed by one more time period

(i). under certain conditions on consumption bundle and the parameters of the

random walk, individual's preferences would be dynamically inconsistent when two consumption values (c and $c + \beta$) fall short of her current reference level but still exceed her pessimistic reference level of next time period, as long as her reference level has same chance to decrease and has less than half chance to increase;

(ii). under certain conditions on consumption bundle and the parameters of the random walk, individual's preference would be dynamically inconsistent when two consumption values (c and $c + \beta$) exceed her current reference level but still fall short of her optimistic reference level of next time period, as long as her reference level has more chance to decrease rather than increase.

One can find more complete text of the proposition in the Appendix C.

Intuitively, when the two consumption values are between the individual's pessimistic reference level of tomorrow and optimistic reference level of tomorrow, her intertemporal preferences over these two consumption values would change when both of them are delayed for one more time period, for some range of parameters of the random walk, hence her intertemporal preferences are dynamically inconsistent.

Notice that dynamic inconsistency over time could not be explained by exponential

discounted utility model (see Section 2.1 of Chapter I). In fact that, the standard exponential discounted utility theory only predicts the individual will have consistent preference over time (see Section 2.1 of Chapter I).

4.2 Intertemporal Trade-off: Choosing Tomorrow

For the sake of completeness, consider a situation, where decision will be made tomorrow rather than today. Therefore the pair of options A and option B are irrelevant anymore, and she will only choose between the amount of consumption value c tomorrow but nothing for the day after, and the other amount of consumption value $c + \beta$ the day after tomorrow but tomorrow. Then, these two intertemporal options are: option C: $(c, 0)$ and D: $(0, c + \beta)$. Following the random walk process, there are three possible realizations of her reference level of tomorrow, which are $r_2 = r_1 + \alpha$, $r_2 = r_1$, and $r_2 = r_1 - \alpha$. Her propose is to maximize her expected utility conditional on her realized reference level. Conditional on each realized reference level, the individual would prefer option C if her realized reference levels of tomorrow are either $r_2 = r_1$, or $r_2 = r_1 - \alpha$, but may prefer option D if her realized reference levels of tomorrow is $r_2 = r_1 + \alpha$. Therefore, when the open option is available, her preferences over the option C and option D are inconsistent with her previous decision when those are made today, where the open option of making

decision tomorrow is not available. This may be one of reasons why people sometimes prefer commitments as a tool of self-control, by which an individual would avoid to change her mind¹³. On the other hand, the open option of making decision tomorrow may make an individual be better off¹⁴. The general calculations of results can be found in Appendix D. Here, only a summary is presented.

Proposition 4. When the realized reference level of tomorrow is either her pessimistic reference level or same as the initial reference level, the individual's would always have present bias, as long as parameters in the same range of inconsistent preferences as that in Proposition 3.

One can find more complete text of the proposition in the Appendix D.

In a summary, in the neighbourhood of her initial reference level, her preferences over two consumption values c and $c + \beta$, would change when both of them are delayed by one more time period, which is dynamically inconsistent. In addition, when an open option of making decision tomorrow is available, her preferences over those two consumption values would be different from those of today when her realized reference level are either equal to or lower than her previous reference level.

If her realized reference levels are higher than her previous one, the open option may

¹³ For example of discussion of temptation, commitment, and self-control see Gul and Pesendorfer (2001, 2004, and 2005).

¹⁴ For example of discussion of choosing among opportunity sets and preference for flexibility see Kreps (1979).

make her better off. Therefore, the individual's intertemporal preferences are dynamically inconsistent, and that is driven by the random walk process of reference levels and loss aversion.

5. Summary and Further Directions

In this chapter, an intertemporal reference-dependent model, in which an individual's reference levels follow a random walk process, may offer an alternative explanation of the present bias as well as the negative time preference (or future bias), (see Loewenstein and Prelec (1991), Lowenstein and Sicherman (1991)). When the consumption value is higher than the individual's pessimistic reference level of tomorrow, she would have present bias as long as the probability of decreasing her reference level of tomorrow is higher than the probability of increasing her reference level of tomorrow. On the other hand, the individual would have future bias when the consumption value is higher than her reference level of today but the probability of decreasing her reference level of tomorrow is lower than the probability of increasing her reference level of tomorrow.

Furthermore, in this chapter, the intertemporal reference-dependent model also

predicts that when an individual faces two consumption values c and $c + \beta$, which are in the neighbourhood of her reference level of today, her preferences over them would change when both of them are delayed by one more time period. When these two consumption values are lower than her reference level of today, her preferences will be dynamically inconsistent as long as the probability of decreasing her reference level of tomorrow is not zero and the probability of increasing her reference level of tomorrow is lower than half. On the other hand, when these two consumption values are higher than her reference level of today, her preference will be dynamically inconsistent as long as the probability of decreasing her reference level of tomorrow is higher than the probability of increasing her reference level of tomorrow.

When an open option of making decision tomorrow is available, as long as her realized reference level of tomorrow is lower than or equal to her reference level of today, she would always prefer the small and sooner intertemporal option to the large but later intertemporal option, which is consistent with her preferences under the circumstance of two time periods. Therefore she would have present bias as long as her realized reference level of tomorrow is lower than or equal to that of today.

In this chapter, an individual's reference levels follow a random walk process, but this is not the only way that reference levels are formed. Alternatively, the

individual's reference levels could endogenously depend upon past choices and experiences¹⁵. This is the subject of future research.

¹⁵ For example of discussion of reference levels depend upon past experiences in two time periods, see Bowman, Minehart, and Rabin (1999).

A Survey-Based Within-Subject Choice Experiment on Intertemporal Decision-Making¹⁶

Abstract: In this chapter, I explore whether subjects' behaviour in a survey-based within-subject choice experiment is consistent with a number of existing and emerging theories, namely hyperbolic discounting theory, Rubinstein's similarity procedure, and a novel intertemporal reference-dependent model, presented in Chapter II. The main result of the survey-based within-subject choice experiment is that none of these three theories explain subjects' behaviour. However, I do find that, among these three theories, the novel intertemporal model of reference-dependent preferences performs no worse than Rubinstein's similarity procedure, and hyperbolic discounting theory performs the worst with subjects' behaviour on subset of questions. Moreover, I could reject the hypothesis that the subjects made their choices at random. Finally, I also found that their behaviour is not consistent with Independence of Irrelevant Alternatives.

¹⁶ I would like to thank the Edinburgh Campaign for financial support on this experiment, Santiago Sánchez-Pagés and Marco Faravelli for useful and constructive suggestions, and Eva Alevyzaki, Alla Doubrovina, Sascha Mohr, Duncan Whitehead, Zhewei Wang, and Haibo Zhang for help to run the experiment.

1. Introduction

Intertemporal decision-making involves tradeoffs between consumptions and savings occurring at different time period, and affects not only an individual's wealth, happiness but also economic prosperity of every country. While intertemporal decision-making is one of the most important human activities, economists still disagree about what is the best way to model decision-making over time.

As intertemporal decision-making is one of the most important human activities, there have been a large number of experimental studies of choices over time. These experimental studies indicate several systematic violations which question the validity of the standard exponential discounted utility model (see Section 2.2 of Chapter I). These experimental studies indicate that discount rates are not constant over time but decline as if humans had hyperbolic discounting; gains are discounted more than losses; small amount of outcomes are discounted more than large amount ones; delaying a consumption requires more compensation than people are willing to pay to speed it up; and outcomes are discounted as a sequence is different from outcomes are discounted singly.

In this chapter, I explore whether subjects' behaviour in a survey-based

within-subject choice experiment is consistent with a number of existing and emerging theories, namely hyperbolic discounting theory (see Section 3 of Chapter I), Rubinstein's similarity procedure (see Section 5 of Chapter I), and a novel intertemporal reference-dependent model (see Chapter II and Chapter III). The main result of the survey-based within-subject choice experiment is that none of these three theories explain subjects' behaviour. However, I do find that, among these three theories, the novel intertemporal model of reference-dependent preferences performs no worse than Rubinstein's similarity procedure, and hyperbolic discounting theory performs the worst with subjects' behaviour on subset of questions. Moreover, I could reject the hypothesis that the subjects made their choices at random. Finally, I also found that their behaviour is not consistent with Independence of Irrelevant Alternatives.

Although the exponential discounted utility model has dominated economic analysis of intertemporal decision-making since it was introduced by Samuelson (1937), there is a lot of empirical and experimental evidence which is inconsistent with predictions of the exponential discounted utility model (see Thaler (1981), Loewenstein and Prelec (1992), and Frederick, Loewenstein, and O'Donoghue (2002)). For example, in a study of Thaler (1981), an individual would prefer one apple today to two apples tomorrow, and at the same time she would prefer two apples in 51 days to one apple in 50 days from now, which reveals that an individual's time discount rate over

longer time horizons is lower than her time discount rate over short time horizons.

Hyperbolic discounting theory was first studied by Strotz (1956), which offers an alternative model of intertemporal decision-making. After the influential work of Laibson (1997), hyperbolic discounting theory has been widely accepted by economists, and has been applied in a large range of issues (see Frederick, Loewenstein, and O'Donoghue (2002)).

Although hyperbolic discounting theory successfully captures that an individual's preference over time is dynamically inconsistent, its descriptive validity continues to be challenged. In the study of Rubinstein (2003), subjects in each of three experiments reveal their preferences over two pairs of intertemporal options. Rubinstein (2003) argues that subjects' choices are incompatible with hyperbolic discounting theory, but could be explained by the similarity procedure, introduced in Rubinstein (1988).

Reference-dependent preference was first studied by Kahneman and Tversky (1979), in which an individual's preferences are defined over changes of wealth relative to her reference level; individuals care much more about losses than gains; individuals reveal diminishing sensitivity for both gains and losses, which means that an S-shaped utility function that is concave over domain of gains and convex over

domain of losses with a kink at zero. Since the influential work of Kahneman and Tversky (1979), the theory of reference-dependent preferences has been applied in analysis of intertemporal decision-making, for example, see Loewenstein and Prelec (1992), Bowman, Minehart, and Rabin (1999) and Köszegi and Rabin (2007). Based on the idea of loss aversion studied by Kahneman and Tversky (1979), and the idea of mental accounting studied by Thaler (1980, 1985, and 1999), a novel intertemporal model of reference-dependent preferences may offer an alternative model of intertemporal decision-making.

In this chapter, I question whether Rubinstein's similarity procedure is the only viable alternative to the hyperbolic discounting theory. I suggest that the novel intertemporal model of reference-dependent preferences works no worse than similarity-based procedure and better than hyperbolic discounting theory.

This chapter is organized as follows: section 2 is devoted to description of experimental methods, design of the survey-based within-subject choice experiment and experimental hypotheses; experimental data is analyzed in section 3; finally, discussion and conclusion are presented in section 4.

2. Experimental Methods and Hypotheses

Due to the difficulties of interpreting empirical field data, an increasingly popular practice is to run economic experiments. In contrast to collecting field data from the real world, an economic experiment is conducted in an artificial environment, allowing investigator for greater control over external influences on subjects' behaviour.

2.1. A Brief Overview of Experimental Methods

There are two most popular experimental designs used in the experimental study in intertemporal decision-making, which are choice design and matching design.¹⁷

Choice design is the most popular method used in the experimental study in intertemporal decision-making. Subjects are required to choose between a pair of intertemporal options, which typically are a small but early outcome and a larger but later outcome. Although it would be better if subjects are only offered one treatment in each session of experiment, choice design could only reveal an upper or lower bound of the discount rate from each pair of intertemporal options, hence subjects are

¹⁷ If a subject's experimental payoff is not in a monotonic relationship with her behaviour in experiment, then this kind of experiments is called survey. Here, in this section, we consider how questions are presented in experiment (or survey). For discussion of definition of experiment and survey, see Friedman and Cassar (2004).

generally asked to choose over multiple pairs of intertemporal options with different outcomes associated with different dates. For example, Rubinstein (2003) used choice design and reported that subjects' behaviour is inconsistent with predictions of hyperbolic discounting theory. The major problem of choice design with multiple pairs of intertemporal options is the sequence effect¹⁸, also known as anchoring effect or order effect. The sequence effect arises when, for example, a subject is offered two pairs of intertemporal options (questions), and her response to the first question would bias the subject towards giving a similar response to the second one. Although some economists suggested to minimize this effect by offering subjects a series of options, over which subjects' response are opposite to each other, the sequence effect may not be eliminated and, in addition, other problems may arise (see Frederick, Loewenstein, and O'Donoghue (2002)).

Another popular design of experiments on intertemporal decision-making is matching design. In matching design, subjects are required to give an answer which is equal to the given intertemporal option (e.g. £15 today =£ ____ in 30 days). Rather than giving an upper or lower bound of discount rates, subjects' responses reveal the exact discount rate in matching design. For example, Thaler (1981) applied the matching design and revealed that small outcomes are discounted more than large outcomes. Anchoring effect (sequence effect, order effect) is not an issue in matching

¹⁸ Sequence effect describes that, for example, if a subject is offered two treatments (questions) in one session, then response to the first treatment would lead the subject to be more willing to have similar response to the second treatment. See Davis and Holt (1993, p30).

design, which is the major advantage of matching design. The major problem of matching design is that subjects may apply some simple rules rather than carefully think about their time preferences, resulting in the discount rates reported from experimental studies to vary across studies.

In addition to the two most popular experimental designs, rating design is also used in experimental study of intertemporal decision-making. In a rating design, each subject is required to rate each intertemporal option in terms of attractiveness or aversiveness. In addition, experimental studies in intertemporal decision-making also employ pricing design, where each subject is required to give her/his willingness to pay to receive some intertemporal options. For example, Loewenstein (1988) applied pricing design and found that people will require more compensation to delay amount of consumption than they are willing to pay to speed it up. Anchoring effect is also a problem of these two experimental designs.

Furthermore, intertemporal options presented in an experiment could involve either real situation or hypothetical situation. Hypothetical situation provide opportunities to elicit subjects' preferences where implementing the real situation is impractical if not impossible, but subjects may not carefully treat hypothetical situation and may not give the same responses as they would do in a real situation. Despite their potential limitations, experiments involving hypothetical situations remain to be an

easier and more popular method of eliciting individual preferences.

2.2. Survey-Based Experiment and Hypotheses

The survey-based within-subject choice experiment was conducted in February of 2007 at the University of Edinburgh. 127 undergraduate students of the University of Edinburgh were invited to participate in the choice experiment. There were six experimental sessions; each session was conducted at the beginning of an economic tutorial. Students in each session were treated as one group and agreed to participate as unpaid volunteers. In question 1 and question 2, subjects are required to choose the favourite choice of each pair of options, and in question 3, subjects are required to rank options from the most favourite one to the least one.

Since the survey-based experiment involved within-subject design, the possibility of the sequence effect (order effect, anchoring effect) was of concern. Hence, in order to focus subjects on each question and reduce the sequence effect between questions, but still be able to study subjects' responses to multi-treatments, questions in the survey-based experiment are offered as following:

Q1. Imagine that you have won a lottery, which is worth £157 that will be received on September 16th 2008. Would you want to delay payment by one day and gain £1

in compensation? Please circle your choice. (Yes or No)¹⁹

Q2. Imagine that you have to choose between the following two options:

Please circle your choice.

- A. Receiving £ 157 on March 17th 2008
- B. Receiving £ 307 on September 17th 2008

Q3. Imagine that you have an investment opportunity on January 2008, which would offer you the following four payoff options for every unit you invest:

- A. Receiving £ 307 on March 17th 2009
- B. Receiving £ 158 on September 17th 2008
- C. Receiving £ 67 on June 16th 2008
- D. Receiving £ 157 on September 16th 2008

Could you please rank these options from the most favourite to the least one?

Notice that, in question 3, option B and option D offer same pair of payments and dates as those offered in question 1.

¹⁹ An informal study supports the hypothesis that there is no significant differences of responses between question 1 and the following question: Imagine that you have to choose between the following two options: Please circle your choice

(A) Receiving £ 157 on September 16th 2008

(B) Receiving £ 158 on September 17th 2008

In addition, there are no significant differences of responses between different orders of options in the above question.

There are three major hypotheses involved in this choice experiment. Hypotheses and predictions of each hypothesis of each question are presented as following:

Hypothesis 1. Subjects' responses are consistent with predictions of hyperbolic discounting theory (see section 3 of Chapter I). Hence, if subjects' responses are consistent with predictions of hyperbolic discounting theory, subjects' responses are predicted as follow:

- (i) Subjects who choose "No" in question 1 would choose "A" in question 2.
- (ii) Subjects who choose "No" in question 1 would rank "D" higher than "B" in question 3.
- (iii) Subjects who choose "Yes" in question 1 would rank "B" higher than "D" in question 3.
- (iv) The theory has no prediction to question 2 when "Yes" is chosen in question 1.

Therefore, hyperbolic discounting theory rules out the following possibilities of combination of choices:

- (i) choosing "No" in question 1 and "B" in question 2;
- (ii) choosing "Yes" in question 1 and ranking "D" higher than "B" in question 3;
- (iii) choosing "No" in question 1 and ranking "B" higher than "D" in question 3.

If subjects' responses are consistent with predictions of hyperbolic discounting

theory (Rubinstein (2003)), subjects will choose “No” in question 1 if the parameter

$\delta_{t^*} < \frac{v(157)}{v(158)}$, where $t^* = 17/09/2008$, but will choose “Yes” otherwise. If subjects’

responses are consistent with predictions of hyperbolic discounting theory, those

subjects, who choose “No” in question 1, should choose option A in question 2.

Therefore, choosing “No” in question 1 but choosing option B in question 2 is

inconsistent with the above described hyperbolic discounting theory.

Proof: (Similar to Rubinstein (2003))

Choosing “No” in question 1 implies that $\delta_{t^*}v(158) - v(158-1) < 0$, where

$t^* = 17/09/2008$. The concavity of v and the fact that the discount rate is

monotonic decreasing imply that $\delta_s v(x) - v(x-1) < 0$ for any amount of money

$x > 158$ and s earlier than September 17th 2008.

$$\begin{aligned}
 & \delta_{t^*} v(307) < v(307-1) \\
 & \delta_{t^*-1} \delta_{t^*} v(307) < \delta_{t^*-1} v(307-2 \times (1)) < \delta_{t^*-180} v(307-2 \times (1)) \\
 & \delta_{t^*-2} \delta_{t^*-1} \delta_{t^*} v(307) < \delta_{t^*-2} \delta_{t^*-180} v(307-3 \times (1)) < \delta_{t^*-181} \delta_{t^*-180} v(307-3 \times (1)) \\
 & \vdots \\
 & \delta_1 \dots \delta_{t^*} v(307) < \delta_1 \dots \delta_{t^*-180} v(307-180 \times (1))
 \end{aligned}$$

Therefore, the straightforward calculation implies that

$$\left(\prod_{s=1,2,\dots,t^*} \delta_s \right) v(307) - \left(\prod_{s=1,2,\dots,t^*-180} \delta_s \right) v(307-180 \times (1)) < 0. \quad \text{In fact that}$$

$307-180 < 157$ completes the proof. Therefore, choosing “No” in question 1 but

choosing option B in question 2 are incompatible not only with the standard exponential discounted utility model but also with hyperbolic discounting theory.

□

Hypothesis 2. Subjects' responses are consistent with predictions of Rubinstein's similarity procedure (see Rubinstein (1988, 2003), and section 5 of Chapter I). The theory has no prediction on each single question, but a subject's preferences of question 1 and question 3 should be consistent with each other. Then subjects' responses are described as following:

- (i) if a subject chooses "No" in question 1, s/he would rank "D" higher than "B" in question 3;
- (ii) if a subject choose "Yes" in question 1, s/he would rank "B" higher than "D" in question 3.

Therefore, Rubinstein's similarity procedure rules out two possibilities of choices:

- (i) choosing "Yes" in question 1 but ranking "D" higher than "B" in question 3;
- (ii) choosing "No" in question 1 but ranking "B" higher than "D" in question 3.

Rubinstein's similarity procedure has no specified prediction on question 1, hence, a subject will choose "No" in question 1, if s/he treat £ 157 and £ 158 as similar amount of money, but the dates September 16th 2008 is preferred to September 17th

2008. Otherwise, s/he will choose “Yes” in question 1. Furthermore, the theory has no specified prediction on question 2 either. Hence, a subject will choose option B in question 2, if s/he treat the two dates in options as similar, but amount of money £ 307 is preferred. Otherwise, s/he will choose option A.

The theory predicts that if a subject chooses “No” in question 1, s/he would rank “D” higher than “B” in question 3; or if a subject choose “Yes” in question 1, s/he would rank “B” higher than “D” in question 3. Therefore, choosing “No” in question 1 but ranking option B higher than option D in question 3, are not only incompatible with both hyperbolic discounting theory and Rubinstein’s similarity procedure but also break the Independence of Irrelevant Alternatives, such that, the utility from an option is unaffected by other options that might be experienced in prior or future periods.

Hypothesis 3. Subjects’ responses are consistent with predictions of the intertemporal reference-dependent model (see Chapter II) where the reference level is choice-set dependent, which is the option offering the earliest date of payment. Then subjects’ responses are described as following:

- (i) choosing “No” in question 1, choosing “B” in question 2, and ranking “B” higher than “D” in question 3 for some range of parameters;
- (ii) choosing “No” in question 1, choosing “A” in question 2, and ranking “B”

higher than “D” in question 3 for a different range of parameters;

(iii) choosing “Yes” in question 1, choosing “B” in question 2, and ranking “B” higher than “D” in question 3 for another different range of parameters .

(Please see appendix E for range calculation).

Therefore, the intertemporal reference-dependent model rules out two possibilities of combination of choices:

- (i) choosing “Yes” in question 1 but choosing “A” in question 2;
- (ii) ranking option “D” higher than option “B” in question 3.

Consider now the intertemporal reference-dependent model, where the reference is the option offering the earliest date of payment. Let us write choice “No” in question 1 as $(157_{16/09/08}, 0_{17/09/08})$, where the first value represents receiving £ 157 on September 16th 2008 and the second value represents receiving £ 0 on September 17th 2008. Similarly, let us write choice “Yes” as $(0_{16/09/08}, 158_{17/09/08})$. Then, if subjects’ responses are consistent with predictions of the intertemporal reference-dependent model, they would choose “No” as long as the inequality $\eta > \frac{150}{157\lambda - 158}$ holds, where $\lambda > \frac{158}{157} > 1$. Otherwise, subjects would choose “Yes” in question 1.

In question 2, the choice-set dependent reference level is “receiving £157 on March 16th 2008”. Similarly, let us write choice A in question 2 as $(157_{16/03/08}, 0_{17/09/08})$, where the first value represents receiving £157 on March 16th 2008 and the second value represents receiving £0 on September 17th 2008. Let us write choice B as $(0_{16/03/08}, 307_{17/09/08})$. Then, if subjects’ responses are consistent with the intertemporal reference-dependent model, they would choose option B, as long as the inequality $\eta < \frac{150}{157\lambda - 307}$ holds, where $\lambda > \frac{307}{157} > 1$. Therefore, as long as the inequality $\frac{1}{157\lambda - 158} < \eta < \frac{150}{157\lambda - 307}$ holds, where $\lambda > 1$, subjects would choose “No” in question 1, but choose option B in question 2.

Similarly, the choice-set dependent reference level is “receiving £67 on June 16th 2008” in question 3, then let us write choice A in question 3 as $(0_{17/06/08}, 0_{16/09/08}, 0_{17/09/08}, 307_{17/03/09})$, choice B as $(0_{17/06/08}, 0_{16/09/08}, 158_{17/09/08}, 0_{17/03/09})$, choice C as $(67_{16/06/08}, 0_{16/09/08}, 0_{17/09/08}, 0_{17/03/09})$, and choice D as $(0_{16/06/08}, 157_{16/09/08}, 0_{17/09/08}, 0_{17/03/09})$. Then, if subjects’ responses are consistent with the intertemporal reference-dependent model, they would rank B higher than option D in question 3.

3. Data Analysis

In this section, I explore whether subject responses are consistent with the hypotheses described in section 3. There are three groups working as control group; group 1, 13 subjects were required to answer question 1; group 2, 18 subjects were required to answer question 2; and group 3, 19 subjects were required to answer question 3. In addition, there were three treatment groups, such that 33 subjects were required to answer both question 1 and question 2 as group 4; 28 subjects were required to answer both question 1 and question 3 as group 5; and 16 subjects were required to answer all three questions as group 6. In the groups of 4, 5, and 6, questions were simultaneously offered to subjects. Table 4 represents the allocation of questions among subjects²⁰.

	Questions	Subjects
Group1	Q1	13
Group2	Q2	18
Group3	Q3	19
Group4	Q1, Q2	33
Group5	Q1, Q3	28
Group6	Q1, Q2, Q3	16
Total		127

Table 4. Allocation of questions

²⁰ The purpose of allocating different questions to different group is to test for reference-dependent preference within questions. If there was reference-dependent preference across questions, answers to, for example, question 1 in group 1 and group 6 would be different.

To make results more comparable, choosing “Yes” in question 1 is interpreted as delaying payment, but choosing “No” is interpreted as not delaying payment; choosing “B” in question 2 is interpreted as delaying payment, but choosing “A” as not delaying payment; ranking “B” higher than “D” in question 3 is interpreted as delaying payment, but ranking “D” higher than “B” is interpreted as not delaying payment.

Table 5 represents subjects’ responses to question 1 from group 1, group 4, group 5, and group 6; Table 6 represents subjects’ responses to question 2 from group 2, group 4, and group 6; and Table 7 represents subjects’ responses to question 3 from group 3, group 5, and group 6.

Q1	G 1	G 4	G 5	G 6	
Delay (Yes)	10	22	16	9	57
No Delay (No)	3	11	12	7	33
	13	33	28	16	90

Table 5. Responses to question 1

Chi-square test for 4 samples in Table 5 shows that there is no statistically significant difference between distributions of responses to question 1, ($\chi^2 = 1.99$, $p = 0.57$).²¹

²¹ For Chi-square and other non-parametric test see Sigel and Castellan (1988). The theoretical Chi-square distribution is a continuous distribution, hence the calculation of the test statistic from observed data is only an approximation to a true Chi-square variable, but this approximation would be good enough as long as each expected (not observed) value in each cell, for example Table 5, is greater than or equal to five.

In addition, Chi-square test for 3 samples in Table 6 shows that there is no statistically significant difference between distributions of responses to question 2, ($\chi^2 = 2.16$, $p = 0.34$).

Q2	G 2	G 4	G 6	
Delay (B)	18	30	14	62
No Delay (A)	0	3	2	5
	18	33	16	67

Table 6. Responses to question 2

	Q3	G 3	G 5	G 6	
Delay	A/B/D/C	11	16	6	33
(B ≻ D)	B/D/A/C	4	3	3	10
	A/C/B/D	0	1	0	1
	A/B/C/D	0	0	1	1
	B/A/D/C	0	1	2	3
	Delay total	15	21	12	48
No	A/D/B/C	4	3	4	11
Delay	D/B/A/C	0	2	0	2
(D ≻ B)	C/D/B/A	0	1	0	1
	A/D/C/B	0	1	0	1
	No delay total	4	7	4	15
Total		19	28	16	63

Table 7. Responses to question 3, the highest rank to the lowest rank are represented from left to right.

Furthermore, Chi-square test for 3 samples in Table 7 shows that there is no statistically significant difference between distributions of responses to question 3 in

terms of delay or no delay, ($\chi^2 = 0.11$, $p = 0.94$)²².

Therefore, in other words, each question is independently treated in group 4, group 5, and group 6 respectively. Such that, for example, delaying payment by choosing “Yes” in question 1 will not lead subject be more willing to delay payment by choosing option B in question 2.

Furthermore, responses to the combination of question 1 and question 2 and responses to the combination of question 1 and question 3 are also analyzed. Table 8 represents responses to question 1 and question 2 in group 4 and group 6. Chi-square test for 2 samples in Table 8 shows that there is no statistically significant difference between distributions of responses to question 1 and question 2, ($\chi^2 = 1.05$, $p = 0.78$).

Q1/Q2	G 4	G 6	
Yes/A	1	1	2
Yes/B	21	8	29
No/A	1	1	2
No/B	10	6	16
	33	16	49

Table 8. Responses to question 1 and question 2 in group 4 and group 6.

In addition, Table 9 represents responses to question 1 and question 3 in group 5 and

²² For discussion of property of transitivity in the question 3 see Mas-Colell, Whinston, and Green (1995, page 7), Tversky et al. (1990), Roelofsma and Read (2000).

group 6. Chi-square test for 2 samples in Table 9 shows that there is no statistically significant difference between distributions of responses to question 1 and question 3, ($\chi^2 = 2.27$, $p = 0.52$).

Q1/Q3		G 5	G 6	
Yes/ $B \succ D$	Yes/A/B/D/C	12	4	16
	Yes/A/C/B/D	1	0	1
	Yes/A/B/C/D	0	1	1
	Yes/B/A/D/C	1	0	1
	Yes/B/D/A/C	2	3	5
Yes/ $B \succ D$ total		16	8	24
Yes/ $D \succ B$	Yes/A/D/B/C	0	1	1
	Yes/ $D \succ B$ total	0	1	1
No/ $B \succ D$	No/A/B/D/C	4	2	6
	No/B/A/D/C	0	2	2
	No/B/D/A/C	1	0	1
	No/ $B \succ D$ total	5	4	9
No/ $D \succ B$	No/D/B/A/C	2	0	2
	No/A/D/B/C	3	0	3
	No/A/D/C/B	1	3	4
	No/C/D/B/A	1	0	1
	No/ $D \succ B$ total	7	3	10
Total		28	16	44

Table 9. Responses to question 1 and question 3 in group 5 and group 6.

Since there is no statistically significant difference among responses to each question in different groups, nonparametric tests are going to focus on changing choices (i.e.

choosing “No” in question 1, choosing “B” in question 2, ranking “B” higher than “D” in question 3) in group 4, group 5, and group 6.

In group 4, 2 subjects changed their choices from delaying payment by choosing “Yes” in question 1 to not delaying payment by choosing option A in question 2; at the same time, 10 subjects changed their choices from delaying payment by choosing option B in question 2 to not delaying payment by choosing “No” in question 1. In addition, in group 6, 1 subject changed his/her choices from delaying payment by choosing “Yes” in question 1 to not delaying payment by choosing option A in question 2; at the same time, 6 subjects changed their choices from delaying payment by choosing option B in question 2 to not delaying payment by choosing “No” in question 1. Table 10 represents the pool of subjects’ responses to question 1 and question 2 from group 4 and group 6.

		Q 1		
		Delay (Yes)	No Delay (No)	
Q 2	No Delay (A)	3	2	5
	Delay (B)	28	16	44
		31	18	49

Table 10. Pool of subjects’ responses to question 1 and question 2 from group 4 and group 6.

3.1. Testing Hyperbolic Discounting Theory

First consider the null hypothesis that subjects' choices are made randomly. That is the probability that a subject will choose "No" (no delay) in question 1 but choose "B" (delay) in question 2 will be the same as the probability that a subject will choose "Yes" (delay) in question 1 but choose "A" (no delay) in question 2. This hypothesis can be rejected, ($\chi^2 = 7.5789$, $p = 0.0059$), in favour of the alternative hypothesis that the probability that a subject will choose "No" (no delay) in question 1 but choose "B" (delay) in question 2 will be higher than the probability that a subject will choose "Yes" (delay) in question 1 but choose "A" (no delay) in question 2.

Result 1. *Hypothesis 1 that subjects' responses are consistent with predictions of hyperbolic discounting theory (see section 2.2 of present chapter) is rejected.*

3.2. Testing Rubinstein's Similarity Procedure

Now consider responses to question 1 and question 3. In group 5, 0 subjects change their choices from delaying payment by choosing "Yes" in question 1 to not delaying payment by ranking option D higher than option B in question 3; at the same time, 5 subjects change their choices from delaying payment by ranking option B higher than option D in question 3 to not delaying payment by choosing "No" in question 1. In addition, in group 6, 1 subject changes his/her choices from delaying payment by

choosing “Yes” in question 1 to not delaying payment by ranking option D higher than option B in question 3; at the same time, 4 subjects change their choices from delaying payment by ranking option B higher than option D in question 3 to not delaying payment by choosing “No” in question 1. Table 11 represents the pool of subjects’ responses to question 1 and question 3 from group 5 and group 6.

		Q 1		
		Delay (Yes)	No Delay (No)	
Q 3	No Delay ($D \succ B$)	1	10	11
	Delay ($B \succ D$)	24	9	33
		25	19	44

Table 11. Pool of subjects’ responses to question 1 and question 3 from group 5 and group 6.

Now consider the null hypothesis that subjects’ choices are made randomly. That is the probability that a subject will choose “No” (no delay) in question 1 but rank “B” higher than “D” (delay) in question 3 will be the same as the probability that a subject will choose “Yes” (delay) in question 1 but rank “D” higher than “B” (no delay) in question 3. This hypothesis can be rejected, $(\chi^2 = 4.9, p = 0.02686)^{23}$, in favour of the alternative hypothesis that the probability that a subject will choose “No” (no delay) in question 1 but rank “B” higher than “D” (delay) in question 3 will be higher than the probability that a subject will choose “Yes” (delay) in question 1 but rank “D” higher than “B” (no delay) in question 3.

²³ If only the cases of that option B and option D are ranked next to each other are considered, then $\chi^2 = 3.15$ and $P = 0.077$.

Result 2. Hypothesis 2 that subjects' responses are consistent with predictions of Rubinstein's similarity procedure (see section 2.2 of present chapter) is rejected.

3.3. Testing the Intertemporal Reference-Dependent Model

To test for the intertemporal reference-dependent model, consider again the pool of responses to question 1 and question 3, which are represented in Table 11. Let us first explore the null hypothesis that subjects' choices are made randomly, so that the probability that a subject will choose "No" (no delay) in question 1 but rank "B" higher than "D" (delay) in question 3 will be the same as the probability that a subject will choose "Yes" (delay) in question 1 but rank "D" higher than "B" (no delay) in question 3. This hypothesis can be rejected, ($\chi^2 = 4.9$, $p = 0.02686$)²⁴, in favour of the alternative hypothesis that the probability that a subject will choose "No" (no delay) in question 1 but rank "B" higher than "D" (delay) in question 3 will be higher than the probability that a subject will choose "Yes" (delay) in question 1 but rank "D" higher than "B" (no delay) in question 3, which is consistent with prediction of the novel intertemporal reference-dependent model with choice-dependent reference point. In addition, proportion of responses consistent

²⁴ If only the cases of that option B and option D are ranked next to each other are considered, then $\chi^2 = 3.15$ and $P = 0.077$.

with the novel intertemporal reference-dependent model with choice-dependent reference point, which is represented in Table 6, should also be considered, since the intertemporal reference-dependent model predicts individuals would always prefer option A to option B to option D but rules out all other possible ranking over these three options in question 3. Importantly, there are only $\frac{35}{63} \approx 55.56\%$ of choices are consistent with the novel intertemporal reference-dependent model.

***Result 3.** Hypothesis 3 that subjects' responses are consistent with predictions of the novel intertemporal reference-dependent model with choice-set dependent reference point (see section 2.2) is rejected.*

In summary, choosing to receive £ 157 on September 16th 2008 in the first question, but choosing to receive £ 307 on September 17th 2008 in the second question is inconsistent with either exponential discounted utility model or hyperbolic discounting theory. Furthermore, ranking the option of receiving £ 158 on September 17th 2008 higher than the option of receiving £ 157 on September 16th 2008 do not only contradict the choice in the first question (choosing to receive £ 157 on September 16th 2008), but is also inconsistent with either the exponential discounted utility model or hyperbolic discounting theory or Rubinstein's similarity procedure. Note that, both hyperbolic discounting theory and Rubinstein's similarity procedure assume that subjects' choice should not be affected by choice set. While,

in contrast, the intertemporal reference-dependent model allows for a possibility that subjects may have different reference level in each question, and allow for effect of choice set on subjects' choice.

3.4. Testing the Relative Performance of the Three Theories

Since each theory rules out some combination of choices, it is important to analyze the proportions of such observations. Table 12 summarize proportions of observations which are inconsistent with each theory.

	Q1/Q2	Q1/Q3	Q1/Q3
HD	No/B	Yes/ $D \succ B$	No/ $B \succ D$
	$\frac{16}{49}$	$\frac{1}{44}$	$\frac{9}{44}$
RS	N/A	Yes/ $D \succ B$	N/ $B \succ D$
	N/A	$\frac{1}{44}$	$\frac{9}{44}$
RD	Yes/A	Yes/ $D \succ B$	N/ $D \succ B$
	$\frac{3}{49}$	$\frac{1}{44}$	$\frac{10}{44}$

Table 12. Inconsistent observations and proportions, where HD stands for Hyperbolic Discounting Theory, RS stands for Rubinstein's Similarity procedure, and RD stands for Intertemporal Reference-Dependent Model.

Importantly, there is no strong winner among these theories. The performance of the three theories for Q1/Q3 comparison is virtually indistinguishable. As for Q1/Q2, hyperbolic discounting theory performs far worse than the intertemporal

reference-dependent model ($16/49 \approx 32.56\%$ of choices are inconsistent with hyperbolic discounting theory versus $3/49 \approx 6.12\%$ of choices are inconsistent with the intertemporal reference-dependent model). However, Q1/Q2 can not be used to analyze relative performances of Rubinstein's similarity procedure versus the intertemporal reference-dependent model, since Rubinstein's similarity procedure allow all possible combination of choices in Q1/Q2 (therefore no specified predictions).²⁵

3.5. Independence of Irrelevant Alternatives

Apart from above described hypotheses and tests – if subjects' responses are consistent with each of the three intertemporal decision-making theories, we should also consider if subjects' responses violate the Independence of Irrelevant Alternatives that the utility from an option is unaffected by other options that might be experienced in prior or future periods.

Hypothesis 4. Subjects' responses are consistent with the Independence of Irrelevant Alternatives. That is, if subjects' behaviours satisfy the of Independence Irrelevant Alternatives, then

(i) if "Yes" is preferred in question 1, the option B should be ranked higher than

²⁵ For an alternative explanation (Vague theory), which does not work for changing choices in Q1/Q3, of combinations of choices in Q1/Q2 see Manzini and Mariotti (2006).

option D in question 3;

(ii) if “No” is preferred in question 1, the option D should be ranked higher than option D in question 3.

Therefore, the Independence of Irrelevant Alternatives rules out possibility of changing choice between question 1 and question 3.

Similar to the discussion in section 3.2 (testing Rubinstein’s similarity), Table 11 represents the pool of subjects’ responses to question 1 and question 3 from group 5 and group 6. The null hypothesis is that subjects’ choices are made randomly. That is the probability that a subject will choose “No” (no delay) in question 1 but rank “B” higher than “D” (delay) in question 3 will be the same as the probability that a subject will choose “Yes” (delay) in question 1 but rank “D” higher than “B” (no delay) in question 3. This hypothesis can be rejected, ($\chi^2 = 4.9$, $p = 0.02686$)²⁶, in favour of the alternative hypothesis that the probability that a subject will choose “No” (no delay) in question 1 but rank “B” higher than “D” (delay) in question 3 will be higher than the probability that a subject will choose “Yes” (delay) in question 1 but rank “D” higher than “B” (no delay) in question 3.

²⁶ If only the cases of that option B and option D are ranked next to each other are considered, then $\chi^2 = 3.15$ and $P = 0.077$.

Result 4. *Hypothesis 4 that subjects' responses are consistent with the Independence of Irrelevant Alternatives is rejected. In addition, the hypothesis that subjects' behaviour is random is rejected as well.*

Obviously, preferring to receive £157 on September 16th 2008 in the first question, but ranking the option of receiving £158 on September 17th 2008 higher than the option of receiving £157 on September 16th 2008 do not only contradict either the exponential discounted utility model or hyperbolic discounting theory or Rubinstein's similarity procedure, but violate the Independence of Irrelevant Alternatives that the utility from an option is unaffected by other options that might be experienced in prior or future periods.

4. Discussion and Conclusion

This survey-based choice experiment involves a within subject design and can be seen as a pilot study that may be of use for a further study in the intertemporal decision-making. Subjects' responses are inconsistent with predictions of hyperbolic discounting theory and Rubinstein's similarity procedure. The experiment results also indicate that reference-dependent preferences could be only one of many

possible reasons why subjects change their choices, because not every subject behaves exactly in the direction of what the intertemporal reference-dependent model predicts. Possibly, individuals' reference points are unobservable, and may vary across individuals. This reflects the weakness of the model of reference-dependent preferences, parameters are very flexible, but there is always a trade-off between a more elaborate model with some flexible parameters and a simple model with a smaller number of parameters. The aim of building up an economic theory is not to just explain a single experiment but to give some predictions about what will happen in the future experiences. Hence further research is needed to pin down these parameters. In addition, future research may also be aimed at eliciting individual-specific reference points. Furthermore, more research can be aimed at comparing relative performance of Rubinstein's similarity procedure versus the intertemporal reference-dependent model.

In conclusion, supported by the analysis of data collected in the survey-based choice experiment, none of the three theories works better than the others in explaining the experimental data. Results of the choice experiment also indicate that the Independence of Irrelevant Alternatives is violated and subjects did not make their choices at random. In addition, results of the survey-based choice experiment also indicate that hyperbolic discounting theory performs the worst among the three theories, and the intertemporal reference-dependent model performs no worse than

the similarity procedure.

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Appendices

Appendix A: Preferences over Two Time Periods

In the intertemporal context, these two intertemporal options could be represented as: option $A: (c, 0)$, and option $B: (0, c)$. The individual's reference levels tomorrow follow the random walk process, thus $\text{prob}(r_2 = r_1 - \alpha) = p$, $\text{prob}(r_2 = r_1) = 1 - p - q$, and $\text{prob}(r_2 = r_1 + \alpha) = q$, where $r_1 - \alpha > 0$, and $\alpha > 0$.

Case 1, where $\frac{c - r_1}{\alpha} \leq -1$. The utility from option A is given by:

$$\begin{aligned} E[U_A(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\ &= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\ &= c - \lambda\eta(2r_1 - c) + (p - q)\lambda\eta\alpha \end{aligned}$$

The utility from option B is given by:

$$\begin{aligned} E[U_B(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\ &= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1 - p - q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 + \alpha)) \\ &= c - \lambda\eta(2r_1 - c) + (p - q)\lambda\eta\alpha \end{aligned}$$

Therefore, $E[U_A] - E[U_B] = 0$ for all parameters of probability.

Case 2, where $-1 < \frac{c - r_1}{\alpha} \leq 0$. The utility from option A is given by:

$$\begin{aligned} E[U_A(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\ &= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1)) \\ &= c - \lambda\eta(2r_1 - c) + (p - q)\lambda\eta\alpha \end{aligned}$$

The utility from option B is given by:

$$\begin{aligned}
 E[U_B(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\
 &= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1 - p - q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 + \alpha)) \\
 &= c - \lambda\eta(2r_1 - c) + (p - q)\lambda\eta\alpha + p(1 - \lambda)\eta(c - r_1 + \alpha)
 \end{aligned}$$

Therefore, $E[U_A] - E[U_B] = p(c - r_1 + \alpha)(\lambda - 1)\eta > 0$ as long as $p > 0$.

Case 3, where $0 < \frac{c - r_1}{\alpha} \leq 1$. The utility from option A is given by:

$$\begin{aligned}
 E[U_A(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\
 &= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
 &= c - \lambda\eta r_1 + \eta(c - r_1) + (p - q)\lambda\eta\alpha
 \end{aligned}$$

The utility from option B is given by:

$$\begin{aligned}
 E[U_B(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\
 &= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1 - p - q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 + \alpha)) \\
 &= c - \lambda\eta r_1 + \eta(c - r_1) + p\eta\alpha - q\lambda\eta\alpha + q(\lambda - 1)\eta(c - r_1)
 \end{aligned}$$

Therefore, $E[U_A] - E[U_B] = p(\lambda - 1)\eta\alpha - q(\lambda - 1)\eta(c - r_1) > 0$ as long as $\frac{p}{q} > \frac{c - r_1}{\alpha}$.

Notice also that $0 < \frac{c - r_1}{\alpha} \leq 1$, hence that $E[U_A] - E[U_B] < 0$ as long as $\frac{p}{q} < \frac{c - r_1}{\alpha} \leq 1$,

such that $p < q$. Hence the inequality $E[U_A] - E[U_B] > 0$ holds as long as $p > q$.

Case 4, where $1 < \frac{c - r_1}{\alpha}$. The utility from option A is given by:

$$\begin{aligned}
 E[U_A(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\
 &= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
 &= c - \lambda\eta r_1 + \eta(c - r_1) + (p - q)\lambda\eta\alpha
 \end{aligned}$$

The utility from option B is given by:

$$\begin{aligned}
 E[U_B(r_1, c_1, r_2, c_2)] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\
 &= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1 - p - q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 + \alpha)) \\
 &= c - \lambda\eta r_1 + \eta(c - r_1) + (p - q)\eta\alpha
 \end{aligned}$$

Therefore, $E[U_A] - E[U_B] = (p - q)(\lambda - 1)\eta\alpha > 0$ as long as $p > q$.

Therefore, under the circumstance of two time periods, her preferences over these two intertemporal options are summarized in Table 13.

	Present Bias $E[U_A] > E[U_B]$	Indifference $E[U_A] = E[U_B]$	Future Bias $E[U_A] < E[U_B]$
$\frac{c - r_1}{\alpha} \leq -1$	N/A	For all parameters	N/A
$-1 < \frac{c - r_1}{\alpha} \leq 0$	$p > 0$	$p = 0$	N/A
$0 < \frac{c - r_1}{\alpha} \leq 1$	$p > q$	$\frac{p}{q} = \frac{c - r_1}{\alpha}$	$p < q$
$1 < \frac{c - r_1}{\alpha}$	$p > q$	$p = q$	$p < q$

Table 13. Intertemporal preferences of delaying one consumption value, where $\text{prob}(r_2 = r_1 - \alpha) = p$, $\text{prob}(r_2 = r_1) = 1 - p - q$, and $\text{prob}(r_2 = r_1 + \alpha) = q$.

Result 1. (Present & Future Bias) Suppose an individual faces choices between A: $(c, 0)$ and B: $(0, c)$ and her preferences are described by (1), (2), and (3) then

(i). (Present bias) When $r_1 - \alpha < c \leq r_1$, the individual would have present bias, so that $E(U_A) > E(U_B)$, as long as $p > 0$, and when $c > r_1$, the individual would have present bias, so that $E(U_A) > E(U_B)$, as long as $p > q$.

(ii). (Future Bias) When $c > r_1$, the individual would have future bias, so that $E(U_A) < E(U_B)$, as long as $p < q$.

(iii). (Indifference) When $c \leq r_1 - \alpha$, the individual would be indifferent between two choices, so that $E(U_A) = E(U_B)$ for all range of parameters.

Appendix B: Preferences over Three Time Periods

In the intertemporal context, these three intertemporal options could be represented as: option A: $(c, 0, 0)$, option B: $(0, c, 0)$, and C: $(0, 0, c)$. There are three possible reference levels tomorrow such that $\text{prob}(r_2 = r_1 - \alpha) = p$, $\text{prob}(r_2 = r_1) = 1 - p - q$, and $\text{prob}(r_2 = r_1 + \alpha) = q$; and there are five possible reference levels the day after tomorrow, thus $\text{prob}(r_3 = r_1 - 2\alpha) = p^2$, $\text{prob}(r_3 = r_1 - \alpha) = 2p(1 - p - q)$, $\text{prob}(r_3 = r_1) = 2pq + (1 - p - q)^2$, $\text{prob}(r_3 = r_1 + \alpha) = 2q(1 - p - q)$, $\text{prob}(r_3 = r_1 + 2\alpha) = q^2$, where $r_1 - 2\alpha > 0$, and $\alpha > 0$.

Case 1, where $\frac{c - r_1}{\alpha} \leq -2$. The utility from option A is given by:

$$\begin{aligned} E[U_A] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\ &= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\ &\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1 - p - q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(0) + g(0|r_1)) \\ &\quad + 2q(1 - p - q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\ &= c - \lambda\eta(3r_1 - c) + 3(p - q)\lambda\eta\alpha \end{aligned}$$

The utility from option B is given by:

$$\begin{aligned} E[U_B] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\ &= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1 - p - q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 + \alpha)) \\ &\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1 - p - q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(0) + g(0|r_1)) \\ &\quad + 2q(1 - p - q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\ &= c - \lambda\eta(3r_1 - c) + 3(p - q)\lambda\eta\alpha \end{aligned}$$

The utility from option C is given by:

$$\begin{aligned}
E[U_C] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
&\quad + p^2(m(c) + g(c|r_1 - 2\alpha)) + 2p(1 - p - q)(m(c) + g(c|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(c) + g(c|r_1)) \\
&\quad + 2q(1 - p - q)(m(c) + g(c|r_1 + \alpha)) + q^2(m(c) + g(c|r_1 + 2\alpha)) \\
&= c - \lambda\eta(3r_1 - c) + 3(p - q)\lambda\eta\alpha
\end{aligned}$$

Therefore, $E[U_A] = E[U_B] = E[U_C]$ for all parameters of probability. In addition,

$$E(U_A) - E(U_B) = 0, \text{ and } E(U_B) - E(U_C) = 0, \text{ therefore } E(U_A) - E(U_B) = E(U_B) - E(U_C)$$

Case 2, where $-2 < \frac{c - r_1}{\alpha} \leq -1$. The utility from option A is given by:

$$\begin{aligned}
E[U_A] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
&\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1 - p - q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(0) + g(0|r_1)) \\
&\quad + 2q(1 - p - q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\
&= c - \lambda\eta(3r_1 - c) + 3(p - q)\lambda\eta\alpha
\end{aligned}$$

The utility from option B is given by:

$$\begin{aligned}
E[U_B] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1 - p - q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 + \alpha)) \\
&\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1 - p - q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(0) + g(0|r_1)) \\
&\quad + 2q(1 - p - q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\
&= c - \lambda\eta(3r_1 - c) + 3(p - q)\lambda\eta\alpha
\end{aligned}$$

The utility from option C is given by:

$$\begin{aligned}
E[U_C] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
&\quad + p^2(m(c) + g(c|r_1 - 2\alpha)) + 2p(1 - p - q)(m(c) + g(c|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(c) + g(c|r_1)) \\
&\quad + 2q(1 - p - q)(m(c) + g(c|r_1 + \alpha)) + q^2(m(c) + g(c|r_1 + 2\alpha)) \\
&= c - \lambda\eta(3r_1 - c) + 3(p - q)\lambda\eta\alpha - p^2(\lambda - 1)\eta(c - r_1 + 2\alpha)
\end{aligned}$$

Therefore, $E[U_A] = E[U_B] > E[U_C]$ as long as $p > 0$. In addition, $E(U_A) - E(U_B) = 0$,

and $E(U_B) - E(U_C) = p^2(\lambda - 1)\eta(c - r_1 + 2\alpha)$, therefore $E(U_A) - E(U_B) < E(U_B) - E(U_C)$ as

long as $p > 0$.

Case 3, where $-1 < \frac{c-r_1}{\alpha} \leq 0$. The utility from option A is given by:

$$\begin{aligned} E[U_A] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\ &= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1-p-q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\ &\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1-p-q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1-p-q)^2)(m(0) + g(0|r_1)) \\ &\quad + 2q(1-p-q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\ &= c - \lambda\eta(3r_1 - c) + 3(p-q)\lambda\eta\alpha \end{aligned}$$

The utility from option B is given by:

$$\begin{aligned} E[U_B] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\ &= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1-p-q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 + \alpha)) \\ &\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1-p-q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1-p-q)^2)(m(0) + g(0|r_1)) \\ &\quad + 2q(1-p-q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\ &= c - \lambda\eta(3r_1 - c) + 3(p-q)\lambda\eta\alpha - p(\lambda-1)\eta(c-r_1+\alpha) \end{aligned}$$

The utility from option C is given by:

$$\begin{aligned} E[U_C] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\ &= m(0) + g(0|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1-p-q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\ &\quad + p^2(m(c) + g(c|r_1 - 2\alpha)) + 2p(1-p-q)(m(c) + g(c|r_1 - \alpha)) + (2pq + (1-p-q)^2)(m(c) + g(c|r_1)) \\ &\quad + 2q(1-p-q)(m(c) + g(c|r_1 + \alpha)) + q^2(m(c) + g(c|r_1 + 2\alpha)) \\ &= c - \lambda\eta(3r_1 - c) + 3(p-q)\lambda\eta\alpha - 2p(1-p-q)(\lambda-1)\eta(c-r_1+\alpha) - p^2(\lambda-1)\eta(c-r_1+2\alpha) \end{aligned}$$

Therefore, the inequality $E[U_A] > E[U_B] > E[U_C]$ as long as $p > 0$ and

$\frac{1-2q}{2q+p-1} > \frac{c-r_1}{\alpha}$. Notice that $-1 < \frac{c-r_1}{\alpha} \leq 0$, for $2q+p-1 > 0$ then the inequality

$E[U_A] > E[U_B] > E[U_C]$ holds, as long as $p > 0$ and $q < \frac{1}{2}$. Furthermore, for

$2q+p-1 < 0$, the inequality $E[U_A] > E[U_B] > E[U_C]$ also holds, as long as $p > 0$ and

$q < \frac{1}{2}$. In addition, $E(U_A) - E(U_B) = p(\lambda-1)\eta(c-r_1+\alpha)$, and

$E(U_B) - E(U_C) = 2p(1-p-q)(\lambda-1)\eta(c-r_1+\alpha) + p^2(\lambda-1)\eta(c-r_1+2\alpha) - p(\lambda-1)\eta(c-r_1+\alpha)$

, therefore $E(U_A) - E(U_B) > E(U_B) - E(U_C)$ as long as $\frac{2q}{2q+p} > \frac{-(c-r_1)}{\alpha}$, where

$\lim_{p \rightarrow 0} \frac{2q}{2q+p} = 1$. Therefore the inequality $E(U_A) - E(U_B) > E(U_B) - E(U_C)$ holds, as long as $-1 < \frac{c-r_1}{\alpha} \leq 0$ where $p > 0$ and $q < \frac{1}{2}$.

Case 4, where $0 < \frac{c-r_1}{\alpha} \leq 1$. The utility from option A is given by:

$$\begin{aligned} E[U_A] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\ &= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1-p-q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\ &\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1-p-q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1-p-q)^2)(m(0) + g(0|r_1)) \\ &\quad + 2q(1-p-q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\ &= c + \eta(c - r_1) - 2\lambda\eta r_1 + 3(p-q)\lambda\eta\alpha \end{aligned}$$

The utility from option B is given by:

$$\begin{aligned} E[U_B] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\ &= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1-p-q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 + \alpha)) \\ &\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1-p-q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1-p-q)^2)(m(0) + g(0|r_1)) \\ &\quad + 2q(1-p-q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\ &= c + \eta(c - r_1) - 2\lambda\eta r_1 + 2(p-q)\lambda\eta\alpha + (p-q)\eta\alpha - q(\lambda-1)\eta(r_1 + \alpha - c) \end{aligned}$$

The utility from option C is given by:

$$\begin{aligned} E[U_C] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\ &= m(0) + g(0|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1-p-q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\ &\quad + p^2(m(c) + g(c|r_1 - 2\alpha)) + 2p(1-p-q)(m(c) + g(c|r_1 - \alpha)) + (2pq + (1-p-q)^2)(m(c) + g(c|r_1)) \\ &\quad + 2q(1-p-q)(m(c) + g(c|r_1 + \alpha)) + q^2(m(c) + g(c|r_1 - 2\alpha)) \\ &= c + \eta(c - r_1) - 2\lambda\eta r_1 + (p-q)\lambda\eta\alpha + 2(p-q)\eta\alpha - q^2(\lambda-1)\eta(r_1 + 2\alpha - c) \\ &\quad - 2q(1-p-q)(\lambda-1)\eta(r_1 + \alpha - c) \end{aligned}$$

Therefore the inequality $E[U_A] > E[U_B] > E[U_C]$ holds, as long as $\frac{p}{q} > \frac{1}{2}$ for $\frac{c-r_1}{\alpha} < \frac{1}{2}$,

and $\frac{p}{q} > \frac{c-r_1}{\alpha}$ for $\frac{c-r_1}{\alpha} > \frac{1}{2}$. Notice also that $0 < \frac{c-r_1}{\alpha} \leq 1$, the inequality $E[U_A] > E[U_B]$ holds as long as $p > q$. Furthermore, notice also that $\frac{1}{2} < \frac{c-r_1}{\alpha} \leq 1$, the inequality $E(U_B) - E(U_C) < 0$ holds, as long as $\frac{1}{2}q < p$. Hence the inequality $E[U_A] > E[U_B] > E[U_C]$ holds, as long as $p > q$ for $0 < \frac{c-r_1}{\alpha} \leq 1$. In addition,

$$E(U_A) - E(U_B) = (p-q)(\lambda-1)\eta\alpha + q(\lambda-1)\eta(r_1 + \alpha - c), \quad \text{and}$$

$$E(U_B) - E(U_C) = (p-q)(\lambda-1)\eta\alpha + q^2(\lambda-1)\eta(r_1 + 2\alpha - c) + 2q(1-p-q)(\lambda-1)\eta(r_1 + \alpha - c) - q(\lambda-1)\eta(r_1 + \alpha - c).$$

Hence, the inequality $E(U_A) - E(U_B) > E(U_B) - E(U_C)$ holds, as long as $\frac{p}{q} > \frac{1}{2}$ when $\frac{c-r_1}{\alpha} < \frac{1}{2}$, and $\frac{p}{q} > \frac{c-r_1}{2(r_1 + \alpha - c)}$ when $\frac{c-r_1}{\alpha} > \frac{1}{2}$. Notice also that $\frac{1}{2} < \frac{c-r_1}{\alpha} \leq 1$, the inequality $1 \leq \frac{\alpha}{c-r_1} < 2$ holds, furthermore the inequality $\frac{q}{p} < \frac{2(\alpha - (c-r_1))}{c-r_1} < 2$. Therefore, the inequality $E(U_A) - E(U_B) > E(U_B) - E(U_C)$ holds, as long as $p > q$ for $0 < \frac{c-r_1}{\alpha} \leq 1$.

Case 5, where $1 < \frac{c-r_1}{\alpha} \leq 2$. The utility from option A is given by:

$$\begin{aligned} E[U_A] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\ &= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1-p-q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\ &\quad + q^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1-p-q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1-p-q)^2)(m(0) + g(0|r_1)) \\ &\quad + 2q(1-p-q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\ &= c + \eta(c - r_1) - 2\lambda\eta r_1 + 3(p-q)\lambda\eta\alpha \end{aligned}$$

The utility from option B is given by:

$$\begin{aligned}
E[U_B] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1 - p - q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 - \alpha)) \\
&+ p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1 - p - q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(0) + g(0|r_1)) \\
&+ 2q(1 - p - q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\
&= c + \eta(c - r_1) - 2\lambda\eta r_1 + 2(p - q)\lambda\eta\alpha + (p - q)\eta\alpha
\end{aligned}$$

The utility from option C is given by:

$$\begin{aligned}
E[U_C] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
&+ p^2(m(c) + g(c|r_1 - 2\alpha)) + 2p(1 - p - q)(m(c) + g(c|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(c) + g(c|r_1)) \\
&+ 2q(1 - p - q)(m(c) + g(c|r_1 + \alpha)) + q^2(m(c) + g(c|r_1 + 2\alpha)) \\
&= c + \eta(c - r_1) - 2\lambda\eta r_1 + (p - q)\lambda\eta\alpha + 2(p - q)\eta\alpha - q^2\eta(\lambda - 1)(r_1 + 2\alpha - c)
\end{aligned}$$

Therefore $E[U_A] > E[U_B] > E[U_C]$ as long as $p > q$. In addition,

$$E(U_A) - E(U_B) = (p - q)(\lambda - 1)\eta\alpha, \quad \text{and}$$

$$E(U_B) - E(U_C) = (p - q)(\lambda - 1)\eta\alpha + q^2(\lambda - 1)\eta(r_1 + 2\alpha - c), \quad \text{therefore}$$

$$E(U_A) - E(U_B) < E(U_B) - E(U_C) \text{ as long as } p > q > 0.$$

Case 6, where $2 < \frac{c - r_1}{\alpha}$. The utility from option A is given by:

$$\begin{aligned}
E[U_A] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
&+ p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1 - p - q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(0) + g(0|r_1)) \\
&+ 2q(1 - p - q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\
&= c + \eta(c - r_1) - 2\lambda\eta r_1 + 3(p - q)\lambda\eta\alpha
\end{aligned}$$

The utility from option B is given by:

$$\begin{aligned}
E[U_B] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1 - p - q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 - \alpha)) \\
&+ p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1 - p - q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(0) + g(0|r_1)) \\
&+ 2q(1 - p - q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\
&= c + \eta(c - r_1) - 2\lambda\eta r_1 + 2(p - q)\lambda\eta\alpha + (p - q)\eta\alpha
\end{aligned}$$

The utility from option C is given by:

$$\begin{aligned}
E[U_C] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
&\quad + p^2(m(c) + g(c|r_1 - 2\alpha)) + 2p(1 - p - q)(m(c) + g(c|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(c) + g(c|r_1)) \\
&\quad + 2q(1 - p - q)(m(c) + g(c|r_1 + \alpha)) + q^2(m(c) + g(c|r_1 + 2\alpha)) \\
&= c + \eta(c - r_1) - 2\lambda\eta r_1 + (p - q)\lambda\eta\alpha + 2(p - q)\eta\alpha
\end{aligned}$$

Therefore $E[U_A] > E[U_B] > E[U_C]$ as long as $p > q$. In addition,

$$E(U_A) - E(U_B) = (p - q)(\lambda - 1)\eta\alpha, \text{ and } E(U_A) - E(U_B) = (p - q)(\lambda - 1)\eta\alpha, \text{ therefore}$$

$$E(U_A) - E(U_B) = E(U_B) - E(U_C).$$

Result 2. (Diminishing marginal utility over time) Suppose an individual faces choices A: $(c, 0, 0)$, B: $(0, c, 0)$, and C: $(0, 0, c)$; and her preferences are described by (1), (2), and (3) then the individual's expected lifetime utility would decrease when the consumption level is delayed period by period, so that $E(U_A) > E(U_B) > E(U_C)$, as well as the difference between two consecutive expected utilities, so that $E(U_A) - E(U_B) > E(U_B) - E(U_C)$,

(i). when $r_1 - \alpha < c \leq r_1$ as long as $p_- > 0$ and $p_+ < \frac{1}{2}$.

(ii). when $r_1 < c \leq r_1 + \alpha$ as long as $p_- > p_+$.

In a summary, in the situation of two time periods, an individual's preferences over the above-mentioned two intertemporal options would show as present bias when $-1 < \frac{c - r_1}{\alpha}$ as long as the probability of decreasing her reference level higher than the probability of increasing her reference level, such that $p > q$; or future bias when

$0 < \frac{c - r_1}{\alpha}$ as long as the probability of decreasing her reference level lower than the probability of increasing her reference level, such that $p < q$. Furthermore, in the situation of three time periods, her expected utilities of delaying the given amount of consumption value are decreasing over time as well as differences between two consecutive expected utilities when the given amount of consumption value is in the neighbourhood of her reference level of today, such that $-\left(\frac{2q}{2q+p}\right) < \frac{c - r_1}{\alpha} \leq 1$. Overall, the individual's intertemporal preferences are driven by the random walk process of reference level and loss aversion.

Appendix C: Dynamically Inconsistent Preferences over Two Consumption Values

These two pairs of intertemporal options could be represented as: option $A: (c, 0, 0)$ and $B: (0, c + \beta, 0)$; option $C: (0, c, 0)$ and $D: (0, 0, c + \beta)$, where $\beta > 0$. There are three possible reference levels tomorrow such that $prob(r_2 = r_1 - \alpha) = p$, $prob(r_2 = r_1) = 1 - p - q$, and $prob(r_2 = r_1 + \alpha) = q$; and there are five possible reference levels the day after tomorrow, thus $prob(r_3 = r_1 - 2\alpha) = p^2$, $prob(r_3 = r_1 - \alpha) = 2p(1 - p - q)$, $prob(r_3 = r_1) = 2pq + (1 - p - q)^2$, $prob(r_3 = r_1 + \alpha) = 2q(1 - p - q)$, $prob(r_3 = r_1 + 2\alpha) = q^2$, where $r_1 - 2\alpha > 0$, and $\alpha > 0$.

Case 1, where $-1 < \frac{c-r_1}{\alpha} < \frac{c+\beta-r_1}{\alpha} \leq 0$. The utility from option A is given by:

$$\begin{aligned}
E[U_A] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1-p-q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
&\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1-p-q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1-p-q)^2)(m(0) + g(0|r_1)) \\
&\quad + 2q(1-p-q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\
&= c - \lambda\eta(3r_1 - c) + 3(p-q)\lambda\eta\alpha
\end{aligned}$$

The utility from option B is given by:

$$\begin{aligned}
E[U_B] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(c+\beta) + g(c+\beta|r_1 - \alpha)) + (1-p-q)(m(c+\beta) + g(c+\beta|r_1)) \\
&\quad + q(m(c+\beta) + g(c+\beta|r_1 + \alpha)) + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1-p-q)(m(0) + g(0|r_1 - \alpha)) \\
&\quad + (2pq + (1-p-q)^2)(m(0) + g(0|r_1)) + 2q(1-p-q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\
&= c - \lambda\eta(3r_1 - c) + 3(p-q)\lambda\eta\alpha - p(\lambda-1)\eta(c-r_1 + \alpha) + \beta + \lambda\eta\beta - p(\lambda-1)\eta\beta
\end{aligned}$$

The utility from option C is given by:

$$\begin{aligned}
E[U_C] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1-p-q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 + \alpha)) \\
&\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1-p-q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1-p-q)^2)(m(0) + g(0|r_1)) \\
&\quad + 2q(1-p-q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\
&= c - \lambda\eta(3r_1 - c) + 3(p-q)\lambda\eta\alpha - q(\lambda-1)\eta(c-r_1 + \alpha)
\end{aligned}$$

The utility from option D is given by:

$$\begin{aligned}
E[U_D] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
&\quad + p^2(m(c + \beta) + g(c + \beta|r_1 - 2\alpha)) + 2p(1 - p - q)(m(c + \beta) + g(c + \beta|r_1 - \alpha)) \\
&\quad + (2pq + (1 - p - q)^2)(m(c + \beta) + g(c + \beta|r_1)) + 2q(1 - p - q)(m(c + \beta) + g(c + \beta|r_1 + \alpha)) \\
&\quad + q^2(m(c + \beta) + g(c + \beta|r_1 + 2\alpha)) \\
&= c - \lambda\eta(3r_1 - c) + 3(p - q)\lambda\eta\alpha - 2p(1 - p - q)(\lambda - 1)\eta(c - r_1 + \alpha) - p^2(\lambda - 1)\eta(c - r_1 + 2\alpha) \\
&\quad + \beta + \lambda\eta\beta - 2p(\lambda - 1)\eta\beta - p^2(\lambda - 1)\eta\beta
\end{aligned}$$

Therefore $E(U_A) > E(U_B)$ but $E(U_C) < E(U_D)$ when

$$\begin{aligned}
&\frac{p[(1 - p - 2q)(c - r_1) - (1 - 2q)\alpha]}{1 + \lambda\eta - (2 - p - 2q)p(\lambda - 1)\eta}(\lambda - 1)\eta < \beta < \frac{p(c - r_1 + \alpha)}{1 + \lambda\eta - p(\lambda - 1)\eta}(\lambda - 1)\eta \quad \text{as long as} \\
&\frac{1 - 2q}{2q + p - 1} > \frac{c - r_1}{\alpha}. \text{ Notice also that } -1 < \frac{c - r_1}{\alpha} < \frac{c + \beta - r_1}{\alpha} \leq 0, \text{ therefore inequalities} \\
&E(U_A) > E(U_B) \text{ and } E(U_C) < E(U_D) \text{ holds, as long as } p > 0 \text{ and } q < \frac{1}{2}.
\end{aligned}$$

Case 2, where $0 < \frac{c - r_1}{\alpha} < \frac{c + \beta - r_1}{\alpha} \leq 1$. The utility from option A is given by:

$$\begin{aligned}
E[U_A] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
&\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1 - p - q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(0) + g(0|r_1)) \\
&\quad + 2q(1 - p - q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\
&= c + \eta(c - r_1) - 2\lambda\eta r_1 + 3(p - q)\lambda\eta\alpha
\end{aligned}$$

The utility from option B is given by:

$$\begin{aligned}
E[U_B] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(c + \beta) + g(c + \beta|r_1 - \alpha)) + (1 - p - q)(m(c + \beta) + g(c + \beta|r_1)) \\
&\quad + q(m(c + \beta) + g(c + \beta|r_1 + \alpha)) + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1 - p - q)(m(0) + g(0|r_1 - \alpha)) \\
&\quad + (2pq + (1 - p - q)^2)(m(0) + g(0|r_1)) + 2q(1 - p - q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 + 2\alpha)) \\
&= c + \eta(c - r_1) - 2\lambda\eta\alpha + 2(p - q)\lambda\eta\alpha + (p - q)\eta\alpha - q(\lambda - 1)\eta(r_1 + \alpha - c) \\
&\quad + \beta + \eta\beta + q(\lambda - 1)\eta\beta
\end{aligned}$$

The utility from option C is given by:

$$\begin{aligned}
E[U_C] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(c) + g(c|r_1 - \alpha)) + (1 - p - q)(m(c) + g(c|r_1)) + q(m(c) + g(c|r_1 + \alpha)) \\
&\quad + p^2(m(0) + g(0|r_1 - 2\alpha)) + 2p(1 - p - q)(m(0) + g(0|r_1 - \alpha)) + (2pq + (1 - p - q)^2)(m(0) + g(0|r_1)) \\
&\quad + 2q(1 - p - q)(m(0) + g(0|r_1 + \alpha)) + q^2(m(0) + g(0|r_1 - 2\alpha)) \\
&= c + \eta(c - r_1) - 2\lambda\eta\alpha + 2(p - q)\lambda\eta\alpha + (p - q)\eta\alpha - q(\lambda - 1)\eta(r_1 + \alpha - c)
\end{aligned}$$

The utility from option D is given by:

$$\begin{aligned}
E[U_D] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2) + m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) + q(m(0) + g(0|r_1 + \alpha)) \\
&\quad + p^2(m(c + \beta) + g(c + \beta|r_1 - 2\alpha)) + 2p(1 - p - q)(m(c + \beta) + g(c + \beta|r_1 - \alpha)) \\
&\quad + (2pq + (1 - p - q)^2)(m(c + \beta) + g(c + \beta|r_1)) + 2q(1 - p - q)(m(c + \beta) + g(c + \beta|r_1 + \alpha)) \\
&\quad + q^2(m(c + \beta) + g(c + \beta|r_1 + 2\alpha)) \\
&= c + \eta(c - r_1) - 2\lambda\eta\alpha + (p - q)\lambda\eta\alpha + 2(p - q)\eta\alpha - q^2(\lambda - 1)\eta(r_1 + 2\alpha - c) \\
&\quad - 2q(1 - p - q)(\lambda - 1)\eta(r_1 + \alpha - c) + \beta + \eta\beta + q^2(\lambda - 1)\eta\beta + 2q(1 - p - q)(\lambda - 1)\eta\beta
\end{aligned}$$

Therefore $E(U_A) > E(U_B)$ but $E(U_C) < E(U_D)$ when

$$\frac{p(1 + 2q)\alpha - q(1 - 2p - q)(c - r_1)}{1 + \eta + q(2 - 2p - q)(\lambda - 1)\eta}(\lambda - 1)\eta < \beta < \frac{p\alpha - q(c - r_1)}{1 + \eta + q(\lambda - 1)\eta}(\lambda - 1)\eta \quad \text{as long as} \quad \frac{p}{q} > \frac{1}{2} \quad \text{for}$$

$$\frac{c - r_1}{\alpha} < \frac{1}{2}, \quad \frac{p}{q} > \frac{c - r_1}{2(r_1 + \alpha - c)} \quad \text{for} \quad \frac{c - r_1}{\alpha} > \frac{1}{2}. \quad \text{Notice also that} \quad 0 < \frac{c - r_1}{\alpha} < \frac{c + \beta - r_1}{\alpha} \leq 1,$$

therefore inequalities $E(U_A) < E(U_B)$ but $E(U_C) > E(U_D)$ hold as long as $\frac{p}{q} < \frac{c - r_1}{\alpha} \leq 1$,

such that $p < q$. Hence the inequality inequalities $E(U_A) > E(U_B)$ but $E(U_C) < E(U_D)$

hold as long as $p > q$.

Result 3. (Dynamic inconsistency) Suppose an individual faces choices between

A: $(c, 0, 0)$ and B: $(0, c + \beta, 0)$ and choices between C: $(0, c, 0)$ and D: $(0, 0, c + \beta)$;

and her preferences are described by (1), (2), and (3) then

(i). When $-1 < \frac{c - r_1}{\alpha} < \frac{c + \beta - r_1}{\alpha} \leq 0$ the individual's preferences would be

$$E(U_A) > E(U_B) \quad \text{but} \quad E(U_C) < E(U_D) \quad \text{where}$$

$$\frac{(p_- - p_+)(c - r_1) - (1 - 2p_+)\alpha}{X - (2p_- + p_-)} < \beta < \frac{c - r_1 + \alpha}{X - 1} \quad \text{for} \quad X = \frac{1 + \lambda\eta}{p_-(\lambda - 1)\eta}, \quad \text{as long as}$$

$$p_- > 0, \text{ and } p_+ < \frac{1}{2}.$$

(ii). When $0 < \frac{c - r_1}{\alpha} < \frac{c + \beta - r_1}{\alpha} \leq 1$ the individual's preferences would be

$$E(U_A) > E(U_B) \quad \text{but} \quad E(U_C) < E(U_D) \quad \text{where}$$

$$\frac{Y(1 + 2p_+)\alpha - (p_- - p_+)(c - r_1)}{Z + (p_+ + 2p_-)} < \beta < \frac{Y\alpha - (c - r_1)}{Z + 1} \quad \text{for} \quad Y = \frac{p_-}{p_+} \quad \text{and} \quad Z = \frac{1 + \eta}{p_+(\lambda - 1)\eta}, \quad \text{as}$$

long as $p_- > p_+$.

Appendix D: Preferences Based on Realized Reference Levels

When it is tomorrow, the pair of options A and option B are irrelevant anymore; and option C and option are reduced to: option C: $(c, 0)$ and D: $(0, c + \beta)$. There are three possible realizations of her reference level tomorrow, which are $r_2 = r_1 + \alpha$, $r_2 = r_1$, and $r_2 = r_1 - \alpha$, and five possible reference levels the day after tomorrow, thus $\text{prob}(r_3 = r_1 - 2\alpha) = p^2$, $\text{prob}(r_3 = r_1 - \alpha) = 2p(1 - p - q)$, $\text{prob}(r_3 = r_1) = 2pq + (1 - p - q)^2$, $\text{prob}(r_3 = r_1 + \alpha) = 2q(1 - p - q)$, $\text{prob}(r_3 = r_1 + 2\alpha) = q^2$, where $r_1 - 2\alpha > 0$, and $\alpha > 0$.

Case 1, where $-1 < \frac{c - r_1}{\alpha} < \frac{c + \beta - r_1}{\alpha} \leq 0$. The utilities from option C and option D are given by:

When $r_2 = r_1 - \alpha$,

$$\begin{aligned}
E[U_c | r_2 = r_1 - \alpha] &= m(c_2) + g(c_2 | r_2) + E[m(c_3) + g(c_3 | r_3)] \\
&= m(c) + g(c | r_1 - \alpha) + (1 - p - q)(m(0) + g(0 | r_1 - \alpha)) + q(m(0) + g(0 | r_1)) \\
&\quad + p(m(0) + g(0 | r_1 - 2\alpha)) \\
&= c - \lambda\eta(r_1 - \alpha) + \eta(c - r_1 + \alpha) + (p - q)\lambda\eta\alpha
\end{aligned}$$

$$\begin{aligned}
E[U_D | r_2 = r_1 - \alpha] &= m(c_2) + g(c_2 | r_2) + E[m(c_3) + g(c_3 | r_3)] \\
&= m(0) + g(0 | r_1 - \alpha) + p(m(c + \beta) + g(c + \beta | r_1 - 2\alpha)) + (1 - p - q)(m(c + \beta) + g(c + \beta | r_1 - \alpha)) \\
&\quad + q(m(c + \beta) + g(c + \beta | r_1)) \\
&= c - \lambda\eta(r_1 - \alpha) + \eta(c - r_1 + \alpha) + (p - q)\eta\alpha - q(\lambda - 1)\eta(r_1 - c) \\
&\quad + \beta + \eta\beta + q(\lambda - 1)\eta\beta
\end{aligned}$$

Under the circumstance of $r_1 - \alpha < c < c + \beta \leq r_1$, $E(U_c | r_2 = r_1 - \alpha) > E(U_D | r_2 = r_1 - \alpha)$ for

same range of parameters as decision was made in the first time period as long as

$$\frac{(p - q)\alpha + q(r_1 - c)}{1 + \eta + q(\lambda - 1)\eta} \geq \frac{p(c - r_1 + \alpha)}{1 + \lambda\eta - p(\lambda - 1)\eta}. \text{ Notice also that } -1 < \frac{c - r_1}{\alpha} < \frac{c + \beta - r_1}{\alpha} \leq 0,$$

therefore the inequality $\frac{(p - q)\alpha + p_+(r_1 - c)}{1 + \eta + q(\lambda - 1)\eta} \geq \frac{p(c - r_1 + \alpha)}{1 + \lambda\eta - p(\lambda - 1)\eta}$ holds as long as

$$\frac{p(1 - p)}{q} > \frac{(\lambda - 1)\eta}{1 + \lambda\eta} \text{ where } p > 0 \text{ and } q < \frac{1}{2}. \text{ Hence, the inequality}$$

$$E(U_c | r_2 = r_1 - \alpha) > E(U_D | r_2 = r_1 - \alpha) \text{ holds as long as } \frac{p(1 - p)}{q} > \frac{(\lambda - 1)\eta}{1 + \lambda\eta} \text{ where } p > 0 \text{ and}$$

$$q < \frac{1}{2}$$

When $r_2 = r_1$,

$$\begin{aligned}
E[U_c | r_2 = r_1] &= m(c_1) + g(c_1 | r_1) + E[m(c_2) + g(c_2 | r_2)] \\
&= m(c) + g(c | r_1) + p(m(0) + g(0 | r_1 - \alpha)) + (1 - p - q)(m(0) + g(0 | r_1)) + q(m(0) + g(0 | r_1 + \alpha)) \\
&= c - \lambda\eta(2r_1 - c) + (p - q)\lambda\eta\alpha
\end{aligned}$$

$$\begin{aligned}
E[U_D|r_2 = r_1] &= m(c_1) + g(c_1|r_1) + E[m(c_2) + g(c_2|r_2)] \\
&= m(0) + g(0|r_1) + p(m(c + \beta) + g(c + \beta|r_1 - \alpha)) + (1 - p - q)(m(c + \beta) + g(c + \beta|r_1)) \\
&\quad + q(m(c + \beta) + g(c + \beta|r_1 + \alpha)) \\
&= c - \lambda\eta r_1 - (1 - p)\lambda\eta(r_1 - c) - q\lambda\eta\alpha + p\eta(c - r_1 + \alpha) + \beta + (1 - p)\lambda\eta\beta + p\eta\beta
\end{aligned}$$

Under the circumstance of $r_1 - \alpha < c < c + \beta \leq r_1$, $E(U_C|r_2 = r_1) > E(U_D|r_2 = r_1)$ for same range of parameters as decision was made in the first time period.

When $r_2 = r_1 + \alpha$,

$$\begin{aligned}
E[U_C|r_2 = r_1 + \alpha] &= m(c_2) + g(c_2|r_2) + E[m(c_3) + g(c_3|r_3)] \\
&= m(c) + g(c|r_1 + \alpha) + p(m(0) + g(0|r_1)) + (1 - p - q)(m(0) + g(0|r_1 + \alpha)) \\
&\quad + q(m(0) + g(0|r_1 + 2\alpha)) \\
&= c - \lambda\eta(2(r_1 + \alpha) - c) + (p - q)\lambda\eta\alpha
\end{aligned}$$

$$\begin{aligned}
E[U_D|r_2 = r_1 + \alpha] &= m(c_2) + g(c_2|r_2) + E[m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1 + \alpha) + p(m(c + \beta) + g(c + \beta|r_1)) + (1 - p - q)(m(c + \beta) + g(c + \beta|r_1 + \alpha)) \\
&\quad + q(m(c + \beta) + g(c + \beta|r_1 + 2\alpha)) \\
&= c - \lambda\eta(2(r_1 + \alpha) - c) + (p - q)\lambda\eta\alpha + \beta + \lambda\eta\beta
\end{aligned}$$

Under the circumstance of $r_1 - \alpha < c < c + \beta \leq r_1$, $E(U_C|r_2 = r_1 + \alpha) < E(U_D|r_2 = r_1 + \alpha)$ for all range of parameters.

Case 2, where $0 < \frac{c - r_1}{\alpha} < \frac{c + \beta - r_1}{\alpha} \leq 1$. The utilities from option C and option D are given by:

When $r_2 = r_1 - \alpha$,

$$\begin{aligned}
E[U_C|r_2 = r_1 - \alpha] &= m(c_2) + g(c_2|r_2) + E[m(c_3) + g(c_3|r_3)] \\
&= m(c) + g(c|r_1 - \alpha) + p(m(0) + g(0|r_1 - 2\alpha)) + (1 - p - q)(m(0) + g(0|r_1 - \alpha)) \\
&\quad + q(m(0) + g(0|r_1)) \\
&= c - \lambda\eta(r_1 - \alpha) + \eta(c - r_1 + \alpha) + (p - q)\lambda\eta\alpha
\end{aligned}$$

$$\begin{aligned}
E[U_D|r_2 = r_1 - \alpha] &= m(c_2) + g(c_2|r_2) + E[m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1 - \alpha) + p(m(c + \beta) + g(c + \beta|r_1 - 2\alpha)) \\
&\quad + (1 - p - q)(m(c + \beta) + g(c + \beta|r_1 - \alpha)) + q(m(c + \beta) + g(c|r_1)) \\
&= c - \lambda\eta(r_1 - \alpha) + \eta(c - r_1 + \alpha) + (p - q)\eta\alpha + \beta + \eta\beta
\end{aligned}$$

Under the circumstance of $r_1 < c < c + \beta \leq r_1 + \alpha$, $E(U_C|r_2 = r_1 - \alpha) > E(U_D|r_2 = r_1 - \alpha)$

for same range of parameters as decision was made in the first time period as long as

$$\frac{(p - q)\alpha}{1 + \eta} \geq \frac{p\alpha + q(r_1 - c)}{1 + \eta + q(\lambda - 1)\eta}. \text{ Notice also that } 0 < \frac{c - r_1}{\alpha} < \frac{c + \beta - r_1}{\alpha} \leq 1, \text{ the inequality}$$

$$\frac{(p - q)\alpha}{1 + \eta} \geq \frac{p\alpha + q(r_1 - c)}{1 + \eta + q(\lambda - 1)\eta} \text{ holds as long as } p - q > \frac{1 + \eta}{(\lambda - 1)\eta}. \text{ Therefore, the inequality}$$

$$E(U_C|r_2 = r_1 - \alpha) > E(U_D|r_2 = r_1 - \alpha) \text{ holds as long as } p - q > \frac{1 + \eta}{(\lambda - 1)\eta}.$$

When $r_2 = r_1$,

$$\begin{aligned}
E[U_C|r_2 = r_1] &= m(c_2) + g(c_2|r_2) + E[m(c_3) + g(c_3|r_3)] \\
&= m(c) + g(c|r_1) + p(m(0) + g(0|r_1 - \alpha)) + (1 - p - q)(m(0) + g(0|r_1)) \\
&\quad + q(m(0) + g(0|r_1 + \alpha)) \\
&= c - \lambda\eta r_1 + \eta(c - r_1) + (p - q)\lambda\eta\alpha
\end{aligned}$$

$$\begin{aligned}
E[U_D|r_2 = r_1] &= m(c_2) + g(c_2|r_2) + E[m(c_3) + g(c_3|r_3)] \\
&= m(0) + g(0|r_1) + p(m(c + \beta) + g(c + \beta|r_1 - \alpha)) + (1 - p - q)(m(c + \beta) + g(c + \beta|r_1)) \\
&\quad + q(m(c + \beta) + g(c + \beta|r_1 + \alpha)) \\
&= c - \lambda\eta r_1 + \eta(c - r_1) + (p - q)\eta\alpha - q(\lambda - 1)\eta(r_1 + \alpha - c) + \beta + \eta\beta + q(\lambda - 1)\eta\beta
\end{aligned}$$

Under the circumstance of $r_1 < c < c + \beta \leq r_1 + \alpha$, $E(U_C|r_2 = r_1) > E(U_D|r_2 = r_1)$ for same

range of parameters as decision was made in the first time period.

When $r_2 = r_1 + \alpha$,

$$\begin{aligned}
 E[U_C | r_2 = r_1 + \alpha] &= m(c_2) + g(c_2 | r_2) + E[m(c_3) + g(c_3 | r_3)] \\
 &= m(c) + g(c | r_1 + \alpha) + p(m(0) + g(0 | r_1)) + (1 - p - q)(m(0) + g(0 | r_1 + \alpha)) \\
 &\quad + q(m(0) + g(0 | r_1 + 2\alpha)) \\
 &= c - \lambda\eta(2(r_1 + \alpha) - c) + (p - q)\lambda\eta\alpha
 \end{aligned}$$

$$\begin{aligned}
 E[U_D | r_2 = r_1 + \alpha] &= m(c_2) + g(c_2 | r_2) + E[m(c_3) + g(c_3 | r_3)] \\
 &= m(0) + g(0 | r_1 + \alpha) + p(m(c + \beta) + g(c + \beta | r_1)) + (1 - p - q)(m(c + \beta) + g(c + \beta | r_1 + \alpha)) \\
 &\quad + p(m(c + \beta) + g(c + \beta | r_1 + 2\alpha)) \\
 &= c - \lambda\eta(2(r_1 + \alpha) - c) + (p - q)\lambda\eta\alpha - p(\lambda - 1)\eta(c - r_1 + 2\alpha) + \beta + \lambda\eta\beta - p(\lambda - 1)\eta\beta
 \end{aligned}$$

Under the circumstance of $r_1 < c < c + \beta \leq r_1 + \alpha$, $E(U_C | r_2 = r_1 + \alpha) > E(U_D | r_2 = r_1 + \alpha)$

for same range of parameters as decision was made in the first time period as long as

$$\frac{p(c - r_1 + 2\alpha)}{1 + \lambda\eta - p(\lambda - 1)\eta} \geq \frac{p\alpha + q(r_1 - c)}{1 + \eta + q(\lambda - 1)\eta}. \text{ Notice that } 0 < \frac{c - r_1}{\alpha} < \frac{c + \beta - r_1}{\alpha} \leq 1, \text{ the inequality}$$

$\frac{p(c - r_1 + 2\alpha)}{1 + \lambda\eta - p(\lambda - 1)\eta} \geq \frac{p\alpha + q(r_1 - c)}{1 + \eta + q(\lambda - 1)\eta}$ holds as long as $p_- > 1 - 2p_+$. Therefore, the inequality $E(U_C | r_2 = r_1 + \alpha) > E(U_D | r_2 = r_1 + \alpha)$ holds as long as $p > 1 - 2q$ where

$$p > q.$$

Appendix E

In question 1, the endogenous reference level is “receiving £157 on September 16th 2008”. “receiving £157 on September 16th 2008” could be represented as option A: $(157_{16/09/08}, 0_{17/09/08})$, in which the second number represents that the individual will get nothing on September 17th 2008; and “delay payment by one day and gain £1 in compensation” could be represented as option B: $(0_{16/09/08}, 158_{17/09/08})$, in

which the first number captures that the individual will get nothing on September 16th 2008.

Therefore the utility of option A is given by

$$U_A(c, r) = U_A((157_{16/09/08}, 0_{17/09/08}) | (157_{16/09/2008}, 0_{17/09/08})) \\ = m(157) = 157$$

and the utility of option B is given by

$$U_B(c, r) = U_B((0_{16/09/08}, 158_{17/09/08}) | (157_{16/09/08}, 0_{17/09/08})) \\ = m(158) + g(0_{16/09/08} | 157_{16/09/08}) + g(158_{17/09/08} | 0_{17/09/08}) \\ = 158 - \lambda \eta 157 + \eta 158$$

Therefore, an individual would prefer option A to option B as long as

$$\eta > \frac{150}{157 \lambda - 307}, \text{ where } \lambda > \frac{158}{157} > 1.$$

In question 2, the endogenous reference level is “receiving £157 on March 17th 2008”. “receiving £157 on March 17th 2008” could be represented as option A: $(157_{16/03/08}, 0_{17/09/08})$; and “receiving £307 on September 17th 2008” could be represented as option B: $(0_{17/03/08}, 307_{17/09/08})$.

Therefore the utility of option A is given by

$$U_A(c, r) = U_A((157_{17/03/08}, 0_{17/09/08}) | (157_{17/03/08}, 0_{17/09/08})) \\ = m(157) = 157$$

and the utility of option B is given by

$$\begin{aligned}
U_B(c, r) &= U_B((0_{17/03/08}, 307_{17/09/08})(157_{17/03/08}, 0_{17/09/08})) \\
&= m(307) + g(0_{17/03/08} | 157_{17/03/08}) + g(307_{17/09/08} | 0_{17/09/08}) \\
&= 307 - \lambda\eta 157 + \eta 307
\end{aligned}$$

Therefore, an individual would prefer option B to option A, as long as

$$\eta < \frac{150}{157\lambda - 307}, \text{ where } \lambda > \frac{307}{157} > 1.$$

Notice that as long as $\lambda > 1$, the inequality $\frac{1}{157\lambda - 158} < \eta < \frac{150}{157\lambda - 307}$ holds, hence an individual would prefer option A to option B in the first question but prefer option B to option A in the second question.

In question 3, the endogenous reference level is “receiving £67 on June 16th 2008”.

“receiving £67 on June 16th 2008” could be represented as option C:

$(67_{16/06/08}, 0_{16/09/08}, 0_{17/09/08}, 0_{17/03/09})$; “receiving £157 on September 16th 2008” could

be represented as option D: $(0_{16/06/08}, 157_{16/09/08}, 0_{17/09/08}, 0_{17/03/09})$; and “receiving

£158 on September 17th 2008” could be represented as option B:

$(0_{17/06/08}, 0_{16/09/08}, 158_{17/09/08}, 0_{17/03/09})$.

Therefore the utility of option B is given by

$$\begin{aligned}
U_B(c, r) &= U_B((0_{17/06/08}, 0_{16/09/08}, 158_{17/09/08}, 0_{17/03/09})(67_{17/06/08}, 0_{16/09/2008}, 0_{17/09/08}, 0_{17/03/09})) \\
&= m(158) + g(0_{17/06/08} | 67_{17/06/08}) + g(158_{16/09/08} | 0_{16/09/08}) \\
&= 158 - \lambda\eta 67 + \eta 158
\end{aligned}$$

and the utility of option D is given by

$$\begin{aligned}
U_D(c,r) &= U_D((0_{17/06/08}, 157_{16/09/08}, 0_{17/09/08}, 0_{17/03/09})|(67_{17/06/08}, 0_{16/09/2008}, 0_{17/09/08}, 0_{17/03/09})) \\
&= m(157) + g(0_{17/06/08}|67_{17/06/08}) + g(157_{16/09/08}|0_{16/09/08}) \\
&= 157 - \lambda\eta 67 + \eta 157
\end{aligned}$$

Therefore, an individual would prefer option B to option D, which is inconsistent with the result of question 1.